(Part II) Lab-session 3

1) Carry out the following integrations numerically.

$$
\begin{aligned}
I_1 &= \int_1^4 \frac{1}{x} dx = \ln 4 \\
I_2 &= \int_1^2 \exp(x)/x dx \approx 3.059116539, \\
I_3 &= \int_0^\pi \sin(x) x^3 dx = \pi^3 - 6\pi, \\
I_4 &= \int_{-\infty}^{\infty} e^{-(x-5)^2} dx = \sqrt{\pi}
\end{aligned}
$$

a) Write a module to carry out this task which puts the final answer into a cell on the Excel worksheet. Use the trapezoid rule

$$I = \int_a^b f(x)dx \approx \Delta \left[\frac{1}{2}(y_1 + y_{n+1}) + \sum_{i=2}^{n} y_i \right]$$

as an approximation. Perform the computations by separating the integration interval $[a, b]$ into $n = 10$, $n = 100$, $n = 1000$ and $n = 10000$ subintervals.

b) Write a user defined function to carry out this task with input parameters a, b and n. The value returned by the function should be the integral. Use Simpson's one-third rule

$$I = \int_a^b f(x)dx \approx \frac{\Delta}{3} \left[\sum_{i=1,3,5,\ldots}^{n-2} y_i + 4y_{i+1} + y_{i+2} \right]$$

as an approximation. Test your function for various values of n and find a large n', such that the final answer on your worksheet does not change for any $n > n'$ up to an accuracy of 6 decimal places. The value for n' does not have to be precise, just try to find the correct order of magnitude.

2) Use the Excel built-in function Goal Seek to solve numerically the following equations:

$$
\begin{aligned}
110x^2 + 1650x - 40040 &= 0, \\
x^3 - 17x^2 + 71x - 55 &= 0.
\end{aligned}
$$