Geometry & Vectors

Coursework 1

(Hand in the solutions to all questions by Tuesday 27/02/07 16:00)

1) (20 marks) The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis in Euclidean space. A and B are points with position vectors $\overrightarrow{OA} = 6\vec{\imath} + 4\vec{\jmath}$ and $\overrightarrow{OB} = 12\vec{\imath} + 16\vec{\jmath}$.

i) Find the position vector for a point C on the line through A and B, i.e. $C \in \overleftrightarrow{AB}$ such that $AC : CB = 2 : 1$.

ii) Find the position vector for a point D, such that C and D divide AB harmonically. [Two points X and Y on a line $\mathcal{L}$ are said to divide AB harmonically if the ratios $AX : XB$ and $AY : YB$ are the same except for a sign, i.e. there exist two scalars λ and μ such that $AX : XB = \lambda : \mu$ and $AY : YB = -\lambda : \mu$]

2) (10 marks) Let $\vec{e}$ be a unit vector. For any vetor $\vec{v}$ we define a new vector

$$\sigma(\vec{v}) = \vec{v} - 2(\vec{v} \cdot \vec{e})\vec{e}.$$

i) Show that $\sigma(\sigma(\vec{v})) = \vec{v}$.

ii) Show that for any two vectors $\vec{u}$ and $\vec{v}$ the relation $\sigma(\vec{v}) \cdot \vec{u} = \vec{v} \cdot \sigma(\vec{u})$ holds.

iii) Show that $|\sigma(\vec{v})| = |\vec{v}|$.

iv) Take $\vec{e} = \frac{1}{\sqrt{2}}(\vec{\imath} + \vec{\jmath})$ and compute $\sigma(\vec{v})$ for $\vec{v}_1 = 5\vec{\imath} + 4\vec{\jmath}$, $\vec{v}_2 = 2\vec{\imath}$ and $\vec{v}_3 = \vec{\imath} + \vec{\jmath} + \vec{k}$. Use the properties i)-iii) and your explicit examples to give a geometrical interpretation for the transformation $\sigma(\vec{v})$.

3) (10 marks) The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis in Euclidean space. A,B, C, D are points with position vectors

$$\overrightarrow{OA} = 2\vec{\imath} + 3\vec{\jmath} + 5\vec{k},\ \overrightarrow{OB} = \vec{\imath} - \vec{\jmath} + \vec{k},\ \overrightarrow{OC} = \vec{\imath} + \vec{\jmath} - 2\vec{k},\ \overrightarrow{OD} = 2\vec{\imath} + 5\vec{\jmath} + 2\vec{k}$$

i) Show that all points are coplanar.

ii) For all possible lines through A,B, C and D determine which pairs are parallel.

4) (10 marks) An electron with charge q moving with velocity $\vec{v}$ in a magnetic field is subject to the Lorentz force

$$\vec{F} = q(\vec{v} \times \vec{B}),$$

where $\vec{B}$ is the magnetic induction. In three experiments, in which the electron is send into different directions, the force $\vec{F}$ is measured.

Experiment 1:	$\vec{v} = \vec{\imath}$	with	$\vec{F} = q(-4\vec{\jmath} + 2\vec{k})$
Experiment 2:	$\vec{v} = \vec{\jmath}$	with	$\vec{F} = q(4\vec{\imath} - \vec{k})$
Experiment 3:	$\vec{v} = \vec{k}$	with	$\vec{F} = q(-2\vec{\imath} + \vec{\jmath})$

Use these measurements to determine $\vec{B}$. Is it possible to design an experiment such that only one measurement is required to find $\vec{B}$?