Geometry & Vectors

Exercises 2

1) Given are the two scalars $\lambda = 4$, $\mu = -3$ and three vectors

$$\begin{aligned} \vec{u} &= 9\vec{\imath} - 3\vec{\jmath} + \vec{k}, \\ \vec{v} &= 5\vec{\imath} + 2\vec{\jmath} - 6\vec{k}, \\ \vec{w} &= 11\vec{\jmath} + \sqrt{5}\vec{k}, \end{aligned}$$

where the vectors $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ constitute an orthonormal basis.

i) Compute the following expressions

$$\vec{u}\cdot\vec{v}, \quad \lambda(\vec{u}+\vec{v})\cdot\vec{w}, \quad |\vec{v}|^2, \quad \mu(\vec{u}\cdot\vec{v})\vec{w}, \quad |\mu(\vec{u}\cdot\vec{v})\vec{w}|, \quad (42+|\vec{w}|^2/\mu)\vec{u}, \quad \vec{o}\cdot\vec{v}.$$

ii) For the vector $\vec{x} = 2\vec{\imath} + \alpha\vec{\jmath} + \vec{k}$, with $\alpha \in \mathbb{R}$, determine the constant α such that the vector $\vec{x}$ is perpendicular to the vector $\vec{v}$.

2) Consider the two vectors

$$\begin{aligned} \vec{u} &= \gamma\vec{\imath} - 2\vec{\jmath} + 3\vec{k}, \\ \vec{v} &= 2\gamma\vec{\imath} + \gamma\vec{\jmath} - 4\vec{k}. \end{aligned}$$

Find the constant (or constants?) $\gamma \in \mathbb{R}$ such that $\vec{u}$ and $\vec{v}$ are perpendicular.

3) Given the points P_1=(2,5) and P_2=(5,-2). Determine the midpoint on the line segment P_1 P_2.

4) The midpoint of the line segment AB is the point C(-4,-3). The position vector of the point A is $\overrightarrow{OA} = 8\vec{\imath} - 5\vec{\jmath}$. Construct the position vector $\overrightarrow{OB}$.

5) Given the points A=(3,2) and B=(6,8) in a plane. The point C is situated on the line $\overleftrightarrow{AB}$. Construct the position vectors $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ and determine the coordinates of the point C such that

$$\text{AC} : \text{CB} = 1 : 2.$$

6) Given are two non-intersecting lines $\mathcal{L}_1$ and $\mathcal{L}_2$ with line segments A_1B_1 and A_2B_2 and midpoints C_1 and C_2 respectively. Show that

$$\overrightarrow{A_1A_2} + \overrightarrow{B_1B_2} = 2\overrightarrow{C_1C_2}.$$

Solutions to exercises 2

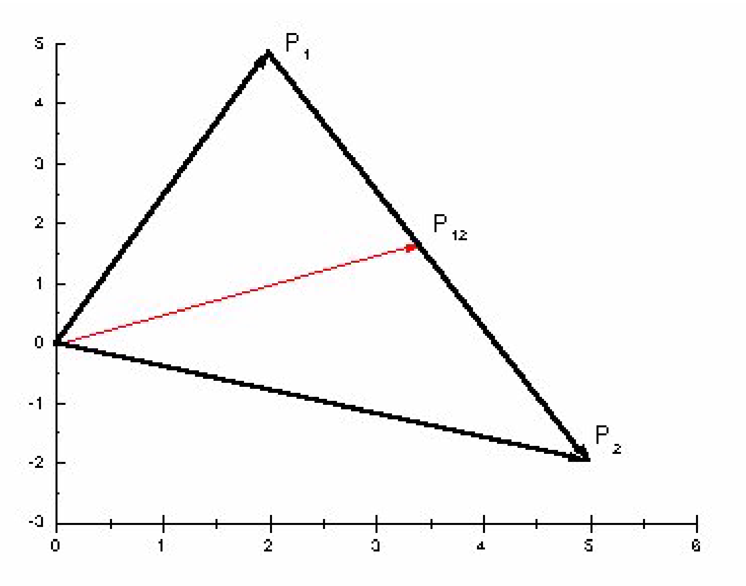
1) i)

$$\begin{aligned}\vec{u}\cdot\vec{v} &= 33, \quad \lambda(\vec{u}+\vec{v})\cdot\vec{w} = -4(11+5\sqrt{5}), \quad |\vec{v}|^2 = 65,\\ \mu(\vec{u}\cdot\vec{v})\vec{w} &= -1089\vec{\jmath} - 99\sqrt{5}\vec{k}, \quad |\mu(\vec{u}\cdot\vec{v})\vec{w}| = 297\sqrt{14}\\ (42+|\vec{w}|^2/\mu)\vec{u} &= \vec{o}, \quad \vec{o}\cdot\vec{v} = 0\end{aligned}$$

ii) $\alpha = -2$.

2) $\gamma = -2$ or $\gamma = 3$.

3)



$$\overrightarrow{OP}_{12} = \overrightarrow{OP}_1 + \frac{1}{2}\overrightarrow{P_1P_2} = \overrightarrow{OP}_1 + \frac{1}{2}(\overrightarrow{OP}_2 - \overrightarrow{OP}_1) \Rightarrow P_{12} = (7/2, 3/2)$$

4) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AC} = \overrightarrow{OA} + 2(\overrightarrow{OC} - \overrightarrow{OA}) \Rightarrow \overrightarrow{OB} = -16\vec{\imath} - \vec{\jmath}$.

5)

$$2\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BC}\right| \quad (1)$$

$$\left|\overrightarrow{AC}\right| + \left|\overrightarrow{BC}\right| = \left|\overrightarrow{AB}\right| = 3\sqrt{5} \quad (2)$$

Make the ansatz $\overrightarrow{OC} = x\vec{\imath} + y\vec{\jmath}$ with unknown constants x, y. Compute with this $\left|\overrightarrow{AC}\right| = \sqrt{(x-3)^2 + (y-2)^2}$ and $\left|\overrightarrow{BC}\right| = \sqrt{(x-6)^2 + (y-8)^2}$. Substitute into (1) and (2) and solve for x, y. Then C=(4,4).

6)

$$\left.\begin{aligned}\overrightarrow{A_1A_2} &= \tfrac{1}{2}\overrightarrow{A_1B_1} + \overrightarrow{C_1C_2} + \tfrac{1}{2}\overrightarrow{B_2A_2}\\ \overrightarrow{B_1B_2} &= -\tfrac{1}{2}\overrightarrow{A_1B_1} + \overrightarrow{C_1C_2} - \tfrac{1}{2}\overrightarrow{B_2A_2}\end{aligned}\right\} \Rightarrow \overrightarrow{A_1A_2} + \overrightarrow{B_1B_2} = 2\overrightarrow{C_1C_2}$$