Solutions CW2 08

i) $\vec{AB} = 3\vec{i} + 6\vec{j} + \vec{k} \quad \Rightarrow \quad \mathcal{L}_1 : \frac{x-1}{3} = \frac{y+1}{6} = \frac{z}{1} = \lambda$

$\vec{CD} = -2\vec{i} + 2\vec{j} - 2\vec{k} \quad \Rightarrow \quad \mathcal{L}_2 : \frac{x-6}{-2} = \frac{y}{2} = \frac{z-3}{-2} = \mu$ (5)

ii) $P_1(1+3\lambda, -1+6\lambda, \lambda) \in \mathcal{L}_1$

$P_1 \in xy\text{-plane} \equiv z = 0 \Rightarrow \lambda = 0 \Rightarrow P_1(1,-1,0) \in xy\text{-plane} \cap \mathcal{L}_1$

$P_2(6-2\mu, 2\mu, 3-2\mu) \in \mathcal{L}_2$

$P_2 \in yz\text{-plane} \equiv x = 0 \Rightarrow \mu = 0 \Rightarrow P_2(0, 6, -3) \in yz\text{-plane} \cap \mathcal{L}_2$ (5)

iii) $3(1+3\lambda) - 4(-1+6\lambda) + \lambda = 21$

$7 - 14\lambda = 21 \Rightarrow \underline{\underline{\lambda = -1}} \Rightarrow P_1(-2, -7, -1) \in \mathcal{L}_1 \cap \mathcal{P}$

$3(6-2\mu) - 4\cdot 2\mu + 3 - 2\mu = 21$

$21 - 16\mu = 21 \Rightarrow \underline{\underline{\mu = 0}} \Rightarrow P_2(6, 0, 3) \in \mathcal{L}_2 \cap \mathcal{P}$ (5) [15

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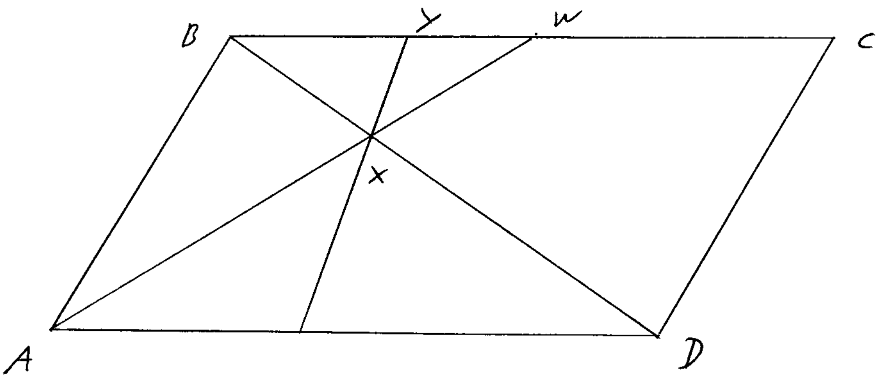

(3)

i) Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be parallel lines in a plane $\mathcal{P}$. M and N are two different lines in the same plane crossing $\mathcal{L}_1$ and $\mathcal{L}_2$ in the points M_1, N_1, M_2, N_2 and intersect in the point X. Then

$$XM_1 : XM_2 = XN_1 : XN_2$$

(2)

ii) With the similarity axiom we read off from the figure

$$\frac{BY}{YC} = \frac{BX}{XD} \qquad \frac{WX}{XA} = \frac{WY}{YB} \qquad \frac{BX}{XD} = \frac{XW}{AX}$$

and $\frac{1}{2} = \frac{BW}{BC} = \frac{BY+YW}{BY+YC} = \frac{1+YW/BY}{1+YC/BY} = \frac{1+WX/XA}{1+XD/BX} = \frac{1+BX/XD}{1+XD/BX} = \frac{BX}{XD}$

$\Rightarrow \qquad \frac{DX}{XB} = 2$

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$\vec{n}_1 = \vec{i} - 4\vec{j} + 9\vec{k}$

$\vec{n}_2 = 2\vec{i} + 3\vec{j} - 5\vec{k}$

$\Rightarrow \vec{n}_1 \times \vec{n}_2 \parallel$ to $\mathcal{L} = \mathcal{P}_1 \cap \mathcal{P}_2$

$\Rightarrow \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -4 & 9 \\ 2 & 3 & 5 \end{vmatrix} = -7\vec{i} + 23\vec{j} + 11\vec{k}$

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We need a point on on $\mathcal{L} = \mathcal{P}_1 \cap \mathcal{P}_2$. It is easy to see

$P(1,0,0) \in \mathcal{L}$

$\Rightarrow \mathcal{L}: \quad \frac{x-1}{-7} = \frac{y}{23} = \frac{z}{11} = \lambda$

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$\left.\begin{array}{l} \vec{AB} = -12\vec{i} + 3\vec{j} - \vec{k} \in \mathcal{P} \\ \vec{AC} = -6\vec{i} + 2\vec{j} - 2\vec{k} \in \mathcal{P} \end{array}\right\} \Rightarrow \vec{n} = \vec{AB} \times \vec{AC}$

$\vec{AX} = (x-5)\vec{i} + (y+1)\vec{j} + (z-1)\vec{k}$

$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -12 & 3 & -1 \\ -6 & 2 & -2 \end{vmatrix} = -4\vec{i} - 18\vec{j} - 6\vec{k}$

$\vec{AX} \cdot (\vec{AB} \times \vec{AC}) = 0 = -4(x-5) - 18(y+1) - 6(z-1) = 0$

$\Rightarrow \quad -4x - 18y - 6z + 8 = 0$

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The plane is unique, as there is just one plane through 3 given noncollinear points (Axiom 2)

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