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Design

ME 1110 – Engineering Practice 1

Engineering Drawing and Design - Lecture 18

Mechanical Elements – Gears

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www.staff.city.ac.uk/~ra600/intro.htm

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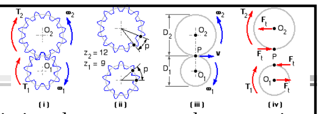
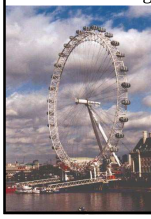
Introduction



- Gears and most of other transmission elements are used to transmit power or to transform rotational movement to translation.
- Gears are most often used in speed reducers:
  - » *Speed* is easy to generate, because voltage is easy to generate
  - » *Torque* is difficult to generate because it requires large amounts of current
- Other driving elements have similar means of action



Dimensions	
Height to top	135 m
Rim diameter	121 m
Hub diameter	4·6 m
Numbers of capsules	32
Numbers of passengers	800
Weights	
Weight of capsules	10 t each
Weight of rim	800 t
Weight of hub/ bearings/spindle	350 t (spindle 200 t)
Weight of A-frame	450 t

LONDON EYE
Tip speed 0.26 m/s
Rotational speed 0.033 rpm {30 min/rev}
Drive power 200 kW !?

Motor speed = 3000 rpm
Motor power = 200 kW
Motor Torque = 640 Nm

$$Pow = \omega T = \frac{n\pi}{30} T$$

Wheel speed = 0.033 rpm
Driving power = 200 kW
Wheel Torque = 600,000,000 Nm

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London eye photographs

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Gear types

Gear	Input/Output		Motion Axis	Loads
Spur	Rotary	Rotary	Parallel	Tangent
Bevel	Rotary	Rotary	Angled	Tangent
Helical	Rotary	Rotary	Parallel or Crossed	Tangent and Axial
Rack	Rotary	Linear	90°	Tangent
Worm	Rotary	Rotary or Linear	90°	Tangent Not back drivable

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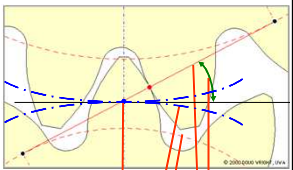
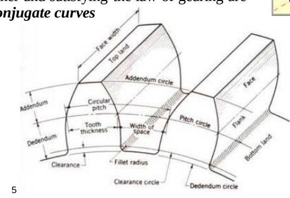
How gears work?

- Law of Gearing:
  - *»A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centres called the **pitch point.***
  - »Any two curves or profiles engaging each other and satisfying the law of gearing are ***conjugate curves***





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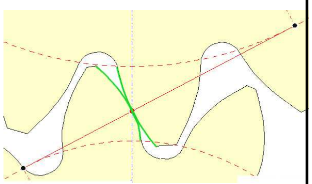
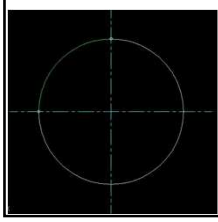
How is a gear tooth formed?

- Involute gears:

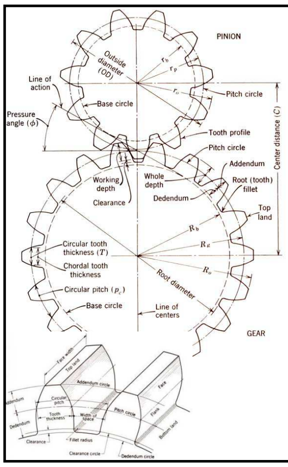
*The fundamental premise of gearing is to maintain a constant relative rotation rate of gears. This can be achieved with a tooth shape called **INVOLUTE.***





Imagine that the pressure line is cut in two.
Trace of the half line end, when the line wraps around the base circle, is the ***involute*** of the ***base circle***.

Gear Parameters



a	Addendum	a (mm) = 25.4/P
a_g	Gear addendum	
a_p	Pinion addendum	
b	Dedendum	b (mm) = 30.48/P+0.05
c	Clearance	c (mm) =5.08/P+.050 (min)
C	Center distance	$C = 0.5(D_P+D_G)$
D	Pitch diameter	$D = N/P = Np/\pi$
D_G	Gear pitch diameter	
D_O	Outside diameter	$D_O = (N+2)/P = D+2a$
D_P	Pinion pitch diameter	
D_B	Base circle diameter	$D_B = D\cos\phi$
D_R	Root diameter	$D_R = D-2b$
ϕ	Pressure angle	
F	Face width (thickness)	
h_k	Working depth of tooth	$h_k = a_g+a_p$
h_t	Whole depth (radial length) of tooth	$h_t = a+b$
$e=1/m_G$	Gear ratio	signΠNinput/ΠNoutput
m	Module (mm only)	m = D/N
N	Number of teeth	N = PD
N_G	Number of teeth on gear	
N_P	Number of teeth on pinion	
p	Circular pitch	$p = \pi D/N = \pi/P$
P	Diametrical pitch (pitch, inches only)	P = N/D
t	Tooth thickness	$t = 0.5\pi/P$

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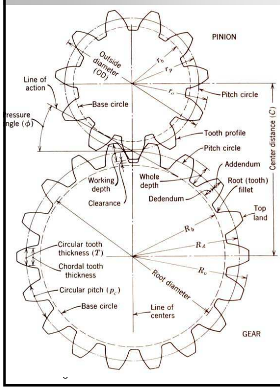
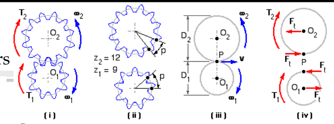
Relations between gear parameters





Gear ratio:

$$P_{ow} = \omega T = \omega_G T_G = \omega_P T_P$$

$$v = \omega_G \frac{D_G}{2} = \omega_P \frac{D_P}{2}$$

$$n_G = \omega_G \frac{\pi}{30};\ n_P = \omega_P \frac{\pi}{30}$$

$$\boxed{GR = \frac{D_G}{D_P} = \frac{N_G}{N_P} = \frac{n_P}{n_G} = \frac{T_G}{T_P}}$$

$$P = \frac{N_G}{D_G} = \frac{N_P}{D_P}$$ Diametrical pitch

$$p = \frac{\pi}{P} = \pi m$$ Circular pitch

$$m = \frac{D}{N} \quad [mm]$$

Module= $\frac{\text{Diameter}}{\text{No. of Teeth}}$

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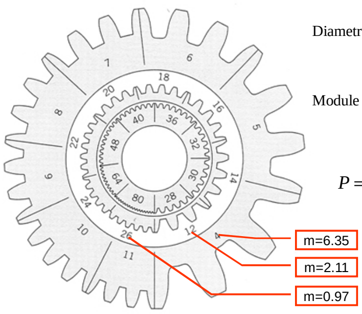
Module and Pitch



Diametrical pitch = $\frac{\text{No. of Teeth}}{\text{Diameter}}$ [in^{-1}]

Module = $\frac{\text{Diameter}}{\text{No. of Teeth}}$ [mm]

$$P = \frac{N}{D}\left[\frac{1}{in}\right];\quad m = \frac{D}{N}[mm]$$

$$m = \frac{25.4}{P}$$

Standard modules are 0.5, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6

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Quiz: gear parameters

Pitch diameter	No of teeth	Diametral pitch [in^{-1}]	Module [mm]
6" (152.4 mm)	72	12	~ 2
90 mm (3.54")	30	~ 8	3
36	12	~ 8	3
125 (4.92")	100	~ 20	1.25

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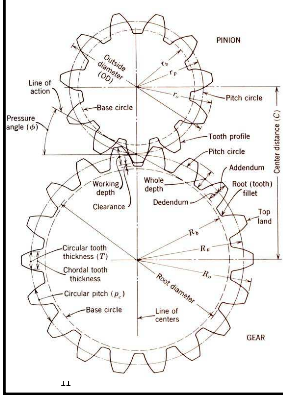
Relations between gear parameters

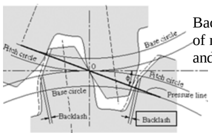


Centre distance: $C = \frac{D_P + D_G}{2}$

Pressure angle: 20° (14.5°)

Addendum: = module
Dedendum: = 1.25 module
Clearance: = 0.25 module

Backlash: clearance measured on the pitch circle of a driving gear



Backlash is function of module and centre distance

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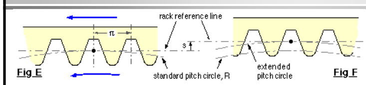
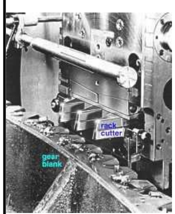
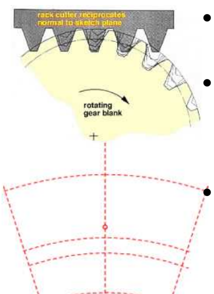
Gear forming – rack generation

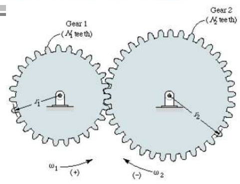






- *Rack* is the gear with infinite radius.
- A rack meshes with a gear in the same way as any other gears mesh.
- A Gear can be formed by a rack cutter commencing two movements:
  - » Reciprocating
  - » Translating
- All gears with the same module are produced by the same rack cutter

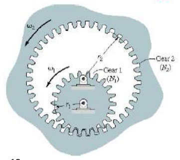
External Meshing



Gear ratio $\boxed{Z_{21} = \frac{\omega_2}{\omega_1} = -\frac{N_2}{N_1}}$

Center distance $\boxed{c_d = r_1 + r_2}$

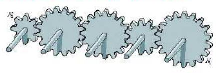
Internal Meshing



Gear ratio $\boxed{Z_{21} = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1}}$

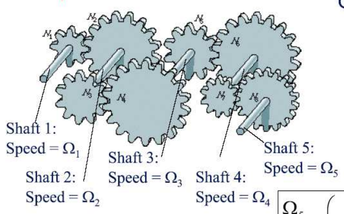
Center distance $\boxed{c_d = r_1 - r_2}$

Simple Gear Trains



Gear ratios $Z_{ji} = \frac{\omega_j}{\omega_i} = \frac{N_i}{N_j}$

Compound Gear Trains



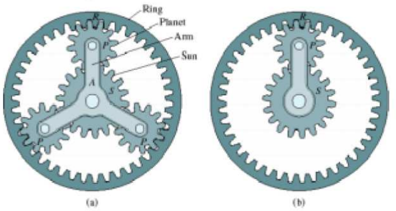
Gear ratio

$$Z_{51} = \frac{\Omega_5}{\Omega_1}$$

$$\frac{\Omega_5}{\Omega_1} = \frac{\Omega_5}{\Omega_4}\frac{\Omega_4}{\Omega_3}\frac{\Omega_3}{\Omega_2}\frac{\Omega_2}{\Omega_1}$$

$$\boxed{\frac{\Omega_5}{\Omega_1} = \left(-\frac{N_7}{N_8}\right)\left(-\frac{N_5}{N_6}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_1}{N_2}\right)}$$

Planetary Gear Train

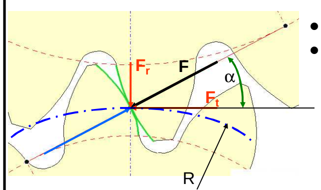


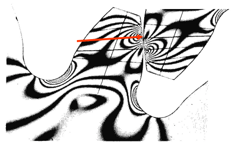
To relate the rpm of the ring to the rpm's of the arm and sun: $\frac{\omega_{ring} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} = -\frac{N_{sun}}{N_{ring}}$

To relate the rpm of the planets to the rpm's of the arm and sun: $\frac{\omega_{planet} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} = -\frac{N_{sun}}{N_{planet}}$

Relationship between the numbers of teeth on the ring, planets and sun: $N_{ring} = N_{sun} + 2N_{planet}$

Gear Force Calculation





- Stress based on the *Force* acting
- The force is caused by the transmitted torque. That force always acts along the pressure line.

$T = P/\omega = F_t R \rightarrow \boxed{F_t = \frac{30P}{\pi n R}[N]}$

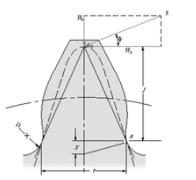
$F = F_t/\cos\alpha \rightarrow \boxed{F = \frac{30P}{\pi n R \cos\alpha}}$

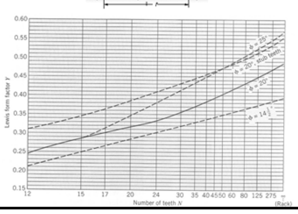
- The force induces stress concentration on gear.

We need to answer:

How much power a pair of gears in question can transfer?

Basic gear stress calculation





- Basic analysis of gear-tooth is based on Lewis Equation which has following assumptions:
  - » The full load is applied on the tip of a single tooth (the worse case)
  - » Radial component is negligible
  - » The load is distributed uniformly along the teeth width
  - » Friction forces are negligible
  - » Stress concentration is negligible.

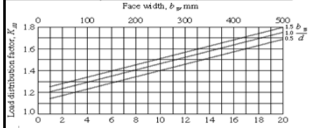
Basic stress in teeth $\sigma_B = \frac{F_t}{mbY}$

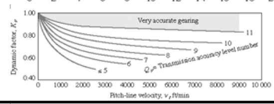
Power transmitted $\boxed{P_B = \frac{S_y}{f_s} n D m b Y}$

Correction factors

Power Source	Driven Machines: Uniform	Light shock	Moderate shock	Heavy shock
	Application factor, K_a			
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Diametral pitch p_d, in.$^{-1}$	Module, m, mm	Size factor, K_s
≥5	≤5	1.00
4	6	1.05
3	8	1.15
3	12	1.25
1.25	20	1.40





- Basic stress must be corrected for:
  - Shocks,
  - Size effects,
  - Uneven load distribution,
  - Dynamic effects.

$$\sigma = \sigma_B \frac{K_a K_s K_m}{K_v}$$

$$P = P_B \frac{K_a K_s K_m}{K_v}$$

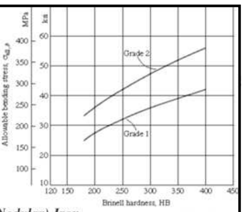
K_a – Application factor

K_s – Size factor

K_m – Load distribution factor

K_v - Dynamic factor

Gear materials



Average Mechanical Properties and Typical Uses of Ductile (Nodular) Iron

Grade[a]	Brinell Hardness, H_B	Elongation (%) (in 50 mm)	Poisson's Ratio	Tensile Modulus GPa	Tensile Modulus 10^6 psi	Typical Uses
60-40-18	167	15.0	0.29	169	24.5	Valves and fittings for steam and chemicals
65-45-12	167	15.0	0.29	168	24.4	Machine components subject to shock and fatigue
80-55-06	192	11.2	0.31	168	24.4	Crankshafts, gears, rollers
120-90-02	331	1.5	0.28	164	23.8	Pinions, gears, rollers, slides

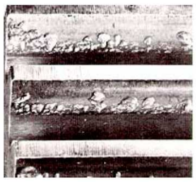
	Tensile Strength				Compressive Strength: Ultimate		Torsional Strength			
	Ultimate		Yield				Ultimate		Yield	
Grade	MPa	10^6 psi	MPa	10^6 psi	MPa	10^6 psi	MPa	10^6 psi	MPa	10^6 psi
60-40-18	461	66.9	329	47.7	359	52.0	472	68.5	195	28.3
65-45-12	464	67.3	332	48.2	362	52.5	475	68.9	297	30.0
80-55-06	559	81.8	362	52.5	386	56.0	504	73.1	193	28.0
120-90-02	974	141.3	864	125.3	920	133.5	875	126.9	492	71.3

[a]The first two sections of grade number indicate minimum values (in ksi) of tensile ultimate and yield strengths.
Source: ASM Metals Reference Book, American Society for Metals, Metals Park, OH, 1981

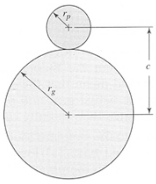
Gear wear

Pitting In Gear Teeth

- **Pitting** – phenomenon in which small particles are removed from the surface of the tooth because of the high contact forces that are present between mating teeth.
- Pitting is actually the fatigue failure of the tooth surface.
- Hardness is the primary property of the gear tooth that provides resistance to pitting.



Example



Pinion A and gear B are shown in figure. Pinion A rotates at 1750 rpm, driven directly by an electric motor. The driven machine is an industrial saw consuming 20 kW. The following conditions are given:

N_P=20	m=3 mm	Q_v=6
N_G=70	b=38 mm	f_s=1.5
n_p=1750 rpm	Pow=20 kW	

What is the centre distance? Compute the stress due to bending in the pinion and gear and find required Brinell hardness for this application.

SOLUTION:

Centre distance:
$$c=\frac{(D_P+D_G)}{2}=m\frac{(N_P+N_G)}{2}=3\frac{90}{2}=135[mm]$$

The pitch diameter of the pinion is:
$$D_P=mN_P=3\cdot 20=60[mm]=0.06[m]$$

The pitch velocity is:
$$\mathrm{v_P}=\frac{\pi \mathrm{n_P D_P}}{60}=\frac{\pi\cdot 1750\cdot 0.06}{60}=5.5[\mathrm{m/s}]=1090[\mathrm{ft/min}]$$

Transferred load (Tangential force):
$$F_t=\frac{60Pow}{\pi n_P D_P}=\frac{60\cdot 20000}{\pi\cdot 1750\cdot 0.060}=3638[N]$$

Example – cont.

From the diagram: Y_P=0.34 and Y_G=0.42

Basic bending stress is: pinion –
$$\sigma_{BP}=\frac{F_t}{mbY_P}=\frac{3638}{0.003\cdot 0.038\cdot 0.34}=94\cdot 10^6[Pa]$$

gear -
$$\sigma_{BG}=\frac{F_t}{mbY_G}=\frac{3638}{0.003\cdot 0.038\cdot 0.42}=76\cdot 10^6[Pa]$$

Correction factors are:
(from diagrams and tables)

Application factor	K_a=1.5
Size factor	K_s=1.0
Load distribution	K_m=1.2
Dynamic factor	K_v=0.68

Corrected pinion bending stress:
$$\sigma_P=\sigma_{BP}\frac{K_aK_sK_m}{K_v}=2.64\cdot 94\cdot 10^6=248[MPa]$$

Allowable stress required for this application:
$$S=f_s\sigma_P=248\cdot 1.5=372[MPa]$$

From the diagram, any material with Brinel hardness higher than HB=400 will satisfy application requirements.