Anglo-Franco-German Representation Theory

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PART I: PREVIOUS TRACK RECORD.

The Principal Investigator (PI). Since September 2011, Principal Investigator Radha Kessar has been a Professor of Mathematics at the University of Aberdeen. After graduating in 1995, the PI held postdoctoral positions in Yale, Minnesota, Oxford, and was an Assistant Professor (tenure-track) at the Ohio State University, before joining the University of Aberdeen in 2005. She has been an invited researcher for the semester long programmes in representation theory at the Bernoulli Centre, EPFL (2005) and at the Mathematical Sciences Research Institute (2008), as well as an Invited Professor at the University of Besancon (June-July 2005). Starting in September 2012, the PI will take up a professorship at City University, London.

The PI has written over thirty five research articles. These have been in representation theory, specifically on the conjectural global-local principles underlying the modular representations of finite groups, and in fusion systems, an area which sits at the cusp of algebraic topology, group theory, and representation theory. Recent work includes the resolution of one direction of R. Brauer's height zero conjecture from 1955 [5], the resolution of a conjecture of K. Erdmann from 1982 [4] and a monograph on fusion systems [6]. In 2009, the PI was awarded the London Mathematical Society's Berwick prize (joint with J. Chuang).

The PI was the main organizer of the 2009 Isle of Skye conference on Algebraic Topology, Group Theory and Representation Theory and since 2010, she has served on the scientific committee of the British Mathematical Colloquium. She has been a subject editor of the Proceedings of the Edinburgh Mathematical Society since 2008, and has been invited to be a guest editor for a special issue of the Journal of Algebra in 2013.

The Co-Investigator (CoI). Co-Investigator Iain Gordon has been The Professor of Mathematics at the University of Edinburgh since 2006, having previously worked at the University of Glasgow. After graduating in 1998, he has had a variety of visiting positions in Edinburgh, Bielefeld, Antwerp, MSRI, University of Washington and the University of California at Santa Barbara. In 2012, he was a senior scientist at the Hausdorff Institute for Mathematics' programme on "Integrability in Geometry and Mathematical Physics" and a guest professor at ETH Zürich.

The CoI is the author of around thirty research articles. These papers are in representation theory, and more specifically in the interactions between it, noncommutative algebra, algebraic geometry and combinatorics. His most recent work is in the representation theory of Cherednik algebras, developing connections with algebraic combinatorics, the geometry of algebraic symplectic varieties, Hecke algebras and noncommutative algebraic geometry, [1], [2], [3]. In 2005 he was

awarded the London Mathematical Society's Berwick Prize, was elected an FRSE in 2010, and was an invited speaker at the last International Congress of Mathematicians in Hyderabad.

The CoI has been involved in much UK and EU collaborative activity. He was a founder and organiser of ARTIN (Algebra and Representation Theory in the North). He has been the organiser of a European Science Foundation training workshop on Derived Categories (2003), a co-organiser of ICMS meetings on "Cherednik Algebras" (2007) and "New Developments in Noncommutative Algebra and its Applications" (2011), "Frontiers of Science" (2008) in Berlin with the Royal Society and the Junge Akademie, and "Categorification and Geometrization", an Isaac Newton Institute sponsored workshop (2009). In 2013 he will be a co-organiser of the MSRI conference on "Interactions between Noncommutative Algebra, Representation Theory, and Algebraic Geometry" and of a conference on "Geometric Representation Theory" in 2014 on Corsica. He is currently serving on the REF2014 mathematics subpanel and from August 2012 will become one of the two editors of the Proceedings of the London Mathematical Society.

Network Partners. At UK level, the proposed network will involve research teams from all major Mathematics Departments, including Aberdeen, Bath, Birmingham, Bristol, Cambridge, East Anglia, Edinburgh, Exeter, Glasgow, Heriot-Watt, Kent, Lancaster, Leeds, Leicester, Liverpool, Imperial College London, King's College London, City University London, Queen Mary University of London, Loughborough, Newcastle, Nottingham, Oxford, Southampton, Sheffield, Swansea, Warwick, York. It will engage leading researchers from a wide spectrum of topics related to representation theory and its connections to neighbouring areas such as algebraic topology, geometry, number theory, computer science and mathematical physics.

In particular, we shall draw on the experience and interactions set up in the framework of the

EPSRC Network EP/F029381/1 "Representation Theory Across The Channel",

which was coordinated by Meinolf Geck (University of Aberdeen) and the CoI from 2008 until the end of 2011. The above network was set up following initial discussions between EPSRC and CNRS, mediated through the British Embassy in Paris in 2006. It was "twinned" with two French partner networks to form a "European Research Partnership" (GDRE 571)[1]; see

`http://math.univ-lyon1.fr/homes-www/remy/GDRE571.html`

for further information about the scope of and the activities supported by this international partnership. In 2009, CNRS and EPSRC took initial steps towards enhancing the original GDRE agreement by approaching the DFG priority programme SPP 1388 "Representation Theory" in Germany[2]. A memorandum for the creation of an international scientific coordination network (GDRI) between the three countries has now been drafted and is under consideration.

[1] See `https://dri-dae.cnrs-dir.fr/spip.php?article1140` for further explanation about such partnerships.

[2] This is documented in emails by Pascal Chossat (CNRS) and David Harman (EPSRC), dated 16 December 2009; see also `http://www.mi.uni-koeln.de/DFG-Schwerpunkt/Schwerpunkt.html`.

PART II: CASE FOR SUPPORT

Representation theory is one of the most vibrant fields of mathematics today. Its history, stretching back to the pioneering work of Frobenius, Burnside and Schur, is a story rich in innovation, implementation of techniques from throughout mathematics, and application to all the sciences. Representation theory is visible on the general mathematical scene through large-scale, semester or even year long research programmes with a distinctive inter-disciplinary flavour, for instance MSRI (Berkeley, USA, 1988, 1990, 2008), Isaac Newton Institute (Cambridge, UK, 1997, 2009), Bernoulli Center (Lausanne, Switzerland, 2005), Institute for Advanced Study (Princeton, USA, 2007–2008), Hausdorff Research Institute for Mathematics (Bonn, Germany, 2011). More spectacularly, recent Fields Medals have been awarded to researchers motivated significantly by representation theory: Lafforgue (2002), Okounkov and Tao (2006), Bào Châu (2010).

A driving force in the development of the subject has always been the abundance of challenging, yet very basic problems, such as finding explicit character formulae for representations. Our view of these problems, however, has undergone a number of profound changes as the field has been revolutionised several times over the last few decades. For instance in the 1970's the introduction of geometric and homological methods led to a flow of new ideas, producing spectacular advances. These ramified far across the subject, heralding in particular the arrival of geometric representation theory. This is typical as we advance the discipline: exciting original fields appear, fields that are of interest well beyond representation theory itself, and we are led to rich new questions.

In tackling some of the fundamental questions and current outstanding problems in representation theory, one can discern the following general, highly interconnected themes.

- **Algebraic Methods in Representation Theory.** Many new representation theoretic structures have appeared recently, including cluster algebras, finite W-algebras and Khovanov-Lauda-Rouquier algebras, and have revolutionised the study of a number of classical objects such as quiver representations, finite group algebras, Hecke algebras and enveloping algebras. The algebraic study of such algebras and their relatives is particularly important, and the combination of classical methods with new approaches arising from cluster mutations and higher Auslander-Reiten theory, graded representation theory, and diagramatic techniques from topology and statistical mechanics, is now producing powerful results.

- **Categorical Methods in Representation Theory.** The use of categorification as a serious tool in representation theory and beyond has blossomed in the last five years, giving rise to "Higher Representation Theory". Here representation theory interacts particularly fruitfully with low dimensional topology, quantum field theory, Lie theory and quantum groups.

- **Cohomological Methods in Representation theory.** Triangulated category and support variety techniques have become an increasingly visible part of representation theory, providing overarching themes for groups, groups schemes, restricted Lie algebras and opening up interactions with commutative algebra and homotopy theory. Powerful results such as the classification of localising subcategories of the non-compact stable module category have been proved recently.

- **Geometric Methods in Representation Theory.** This area is perhaps dominated by the Geometric Langlands' Programme, and through this there are strong connections to mathematical physics and to number theory. In a related direction there are rich links to homological mirror symmetry and algebraic geometry, particularly through the use of derived categories, and there has also been recent remarkable progress in connection with Lie theoretic representation theory in positive characteristic.

- **Topological Methods in Representation Theory.** Interactions with homotopy theory have led to the emergence of algebro-topological structures such as fusion systems, p-local groups and partial groups. These developments have had a significant impact on classical areas such as the theory of finite simple groups and Brauer's theory of modular group representations. There are also strong links to differential topology through the Riemann-Hilbert correspondence, cohomology theories appear in various guises, and many invariants of low dimensional topology have important representation theoretic interpretations.

The UK is a major international player in Representation Theory with leading practitioners throughout the country. For instance a total of 6 members of the previous network spoke at the last two International Congress of Mathematicians; the recent International Review of Mathematics asserted that work in representation theory in the UK is world-leading. To maintain and enhance this strength, international collaboration on a whole range of levels is vital. This applies, in particular, to the training of PhD students and early-career researchers.[3] Representation Theory is characterised by its use of and connection to many diverse areas of mathematics: it is crucial for the next generation of researchers that we provide this breadth, both to allow young representation theorists to gain experience of other areas of mathematics, and to allow talents from other topics to see representation theory in action. The training and opportunities that we intend the network to offer will be broad, but high level, at a scope far beyond anything that can be offered for instance by the Taught Course Centres in the UK.

Our previous grant EP/F029381 and the corresponding GDRE agreement came to an end in December 2011. We are proposing an essentially new network, still partnered with France through the CNRS, but now taking systematic advantage of the strategic opportunity to partner the UK with Germany. The DFG is currently funding a major six-year long priority programme on representation theory worth around 16 million euros which will continue for another three years; there is a keen interest from the German side as well to engage in a formal cooperation with France and the UK. This new, pan-european network will have significant scientific benefits for the UK. Representation theoretic interests in the three countries overlap significantly but there are clear complementary strengths. For instance, the UK and France have considerable expertise in group theory, while France and Germany are particularly strong in Lie theory, with the UK and Germany being masters in representations of algebras. Of course, the research communities in these three countries already linked through a number of individual contacts and collaborations. But we strongly believe that, given the eclectic nature of representation theory, coordinated cooperation in

[3] A recurring theme in the international reviews [7], [8] is that, to a large extent, UK PhDs are not competitive for academic positions in today's global market.

the framework of a network with the appropriate financial support from each partner, will lead to a significantly higher synergy. The resulting breadth and medium-term activity will be of significant benefit to UK representation theorists at all levels.

Thus, we are now in a unique position to achieve a step-change in international cooperation over the next 3 years around the theme of "representation theory", between France, Germany and the UK.

Initial Membership. The main feature of this new network will be a close cooperation and coordination of activities with partner networks and institutions in both France and Germany. At UK level, the proposed network will involve research teams from all major Mathematics Departments. Administratively, this will be done through a GDRI agreement with the following initial members:

- **France:**
  - CNRS, contact: Anne-Marie Brass
  - University Paris 7, Institut de Mathématiques de Jussieu, contact: Jean Michel.
  - GDR "Théorie de Lie Algébrique et Géometrique", coordinated by Cédric Bonnafé (University of Montpellier, France); see

    `http://ens.math.univ-montp2.fr/~bonnafe/GDR/index.html`

- **Germany:**
  - DFG, contact: Dr. Frank Kiefer
  - DFG-Priority Programme SPP 1388 "Representation Theory", coordinated by Peter Littelman (University of Cologne, Germany); see

    `http://www.mi.uni-koeln.de/DFG-Schwerpunkt/Schwerpunkt.html`

- **UK:**
  - EPSRC, contact: Nick Cook;
  - City University, contact: Radha Kessar;
  - University of Edinburgh, contact: Iain Gordon.

The PI will be responsible for the overall **management** of the network working closely with the CoI. The duties of the PI and CoI include:

- coordinating the exchanges between the three partner countries, in close consultation with the representatives of the French and German networks;
- encourage network partners to initiate activities in targeted areas;
- maintaining a webpage containing lists of members with interests, information about forthcoming meetings, links to earlier meetings with active hyperlinks and notes, etc.

Matters of a more administrative nature (e.g., refunding of expenses for travel and subsistence) will be handled through the administration of City University.

A **steering committee** will be set-up in connection to this network, including representatives from the network partners. The steering committee will oversee the scientific content and balance of the network's activities. The current proposal is an academic successor to EPSRC Network/EP/F029381/1, which was co-ordinated by Meinolf Geck and the CoI. Professor Geck is intending to leave for a position in Germany in September 2012, as a result of which Radha Kessar has taken over. However, Professor Geck is willing to continue his association with the new network as a member of the steering commitee.

Objectives. While the internet and electronic communication play an increasingly important role in the exchange of research and ideas, the possibility to meet and discuss individually or in small groups and workshops remains both a key source of information about what is happening at the international level and a key ingredient in the discovery and development of new routes of research. In a somewhat different direction, one of the high-level recommendations of the Panel of the International Review of Mathematics 2010 concerns "continued action and attention directed toward improving the quality of PhD training". Thus, the central objectives of the proposed network are:

- Provide workshop programs and training opportunities for early-career researchers.
- Facilitate and encourage contacts and exchanges between researchers in France, Germany and the UK.
- Facilitate interactions between representation theory and other research areas.

By providing the opportunity for frequent contact and collaboration between researchers at different career levels and from institutions across the three countries, the network will be a key facilitator in shaping the direction of a central area of mathematics. Pursuing these objectives leads to a significant increase in international research collaborations and all the benefits that come with these: short time visits of PhD students, invited lecture series, enhanced scope of mini-events and so on. Numerous examples can be found on the summary page of the GDRE at

`http://math.univ-lyon1.fr/homes-www/remy/GDRE571.html`.

The proposed new network will bring this to an even higher level, by extending the range of activities from France and the UK to Germany. We have one final long-term goal for which the award of this grant is a crucial stepping stone.

- Apply for a european-funded network in Representation Theory, building on the collaborations between France, Germany and the UK.

We intend to apply for this either through the ERC scheme for Marie Curie Initial Training Networks or the Research Networking Programmes of the European Science Foundation.

Activities (2012–2015). The proposed network will mainly support, encourage and coordinate the following activities.

- Support travel of early-career and established researchers to work on specific projects with colleagues overseas, or to attend workshops. This includes, in particular, participation in the relevant annual GDR workshops at the CIRM in France. (The most recent appropriate meeting would have been in April 2012; see `http://njacon.disque.math.cnrs.fr/`.) As a general rule, the proposed network will provide support for subsistence for visitors to the UK and travel support for visits out of UK with the explicit understanding that the French and German partner networks reciprocate, applying analogous rules.

- Organisation of seminar-style research oriented workshops for early-career researchers (on the model of the hugely successful Isle of Skye workshop in 2010; see

  `http://www.abdn.ac.uk/~mth190/skye2010.html`). The workshops will normally have two parts: A working seminar with talks given by participants. The seminar will be focussed on a particular research topic or problem, chosen by network organisers. The second part of the workshop will consist of a few invited talks by researchers on their own results. The aim of this participant driven "total immersion" style is to spur new collaborations around the topic of the conference. We will hold these annually, once in each of the three countries. A preliminary reservation has been made for such an event in 2014 at the Institute d'Etudes Scientifiques de Cargèse on Corsica. In each instance, the hosting country will provide the bulk of the funding for the workshop (including subsistence, venue costs etc.) with the other two partners funding the travel of their members.

- Enhance the scope (audience, speakers, range of topics) and international standing of regional and interregional meetings (e.g., ARTIN, BLOC, Kent Algebra Days), with the added benefit to encourage and initiate new collaborations.

- Run a biannual representation theory seminar focussing on enhancing the research collaborations between representation theory and other topics both within mathematics (e.g. number theory, algebraic geometry) and beyond mathematics (e.g. medicine, quantum computing). This will meet on a rotating principle in Universities in France, Germany and the UK.

Plans for dissemination. We will have a network webpage with an RSS feed which will be the main gateway to information about the network. It will contain lists of members with interests, forthcoming meetings, links to earlier meetings with active hyperlinks and notes, etc. Dissemination will also take place in the proposed conferences, workshops, seminars and research visits. Finally we plan several public interest lectures, noting that some of the mathematicians most visible to the public – Marcus du Sautoy and Colva Roney-Dougal – work in topics very closely related to representation theory.

Potential for collaboration. The key aspect of network is collaboration. The network is designed to facilitate a variety of different formats for collaboration, some of which are novel and others tried and tested, but throughout a crucial ingredient is that the intellectual content will be formulated by the members of the network themselves. Thus topics will emerge in response to new developments in the field as well as the fundamental intellectual needs of the UK community, as understood by

the mathematicians working at the international forefront of representation theory. The role of the PI and CoI, in collaboration with the network partners in France and Germany, will be to make sure that over the term of the grant these topics cohere into a broad and balanced portfolio.

We see particular room for collaboration within the biannual seminar series, where it is our intention to include topics on the applications of representation theory, both within and outwith mathematics. Similarly, access to resources in both France and Germany will increase the possibility for cross-fertilisation with subjects beyond the typical boundaries of representation theory. For instance, it is commonly the case that workshops in Luminy address topics that straddle more than one field, with a recent relevant example being "Variétes de carquois, invariants de Donaldson-Thomas et instantons".

As one of many possible explicit examples of ongoing large-scale international collaborations at the forefront of current research we mention the joint projects of Fiebig (Erlangen, Germany), Juteau (Caen, France), Mautner (Harvard, USA) and Williamson (formerly Oxford, now Bonn, Germany) around "modular perverse sheaves" and their applications to classical problems in representation theory. The previous network grant EP/F029381/1 played a role in supporting this collaboration, both through support for short visits and then by organising the seminar-style workshop on the Isle of Skye in 2010(see `http://www.abdn.ac.uk/∼mth190/skye2010.html`) which provided a forum to present, discuss and develop these ideas, while at the same giving PhD students and early-career researchers the opportunity to participate in this process. The 2010 workshop mentioned above was an experiment, and it turned out to be hugely successful. One of the purposes of this new proposal is to build on this experience and encourage and support further collaborations of this kind.

National Importance. Mathematical Sciences underpin our 21st century technology, economy and society, and serve as one of the pillars of education at all levels. Major contributions to the health and prosperity of society arise from insights, results and algorithms created by the entire sweep of the mathematical sciences, ranging across the purest of the pure, theory inspired by applications and hands-on applications. This has been the case already with the Ancient Greeks, and it is as true as ever now.

Representation Theory, understood in the broad sense of this proposal, is a key area of mathematical sciences, with multiple and complex connections to neighboring sciences like Physics (e.g., elementary particle theory, quantum computing, random matrices, string theory), Biology and Chemistry (e.g., symmetry in molecular structures). It is a rich area of research in its own right, touching upon fundamental and deep mathematical problems, and, through its wide range of connections, it is a key to maintaining the health of other research disciplines.

References

[1] I. Gordon, On the quotient ring by diagonal invariants, *Invent. Math.* **153** (2003), no. 3, 503–518.

[2] I. Gordon and J.T. Stafford, Hilbert schemes and rational Cherednik algebras I & II, *Adv. Math.* **198** (2005), no. 1, 222–274; *Duke. Math J.* **132** (2006), no. 1, 73–135.

[3] I. Gordon, Macdonald positivity via the Harish-Chandra D-module, *Invent. Math.* **187** (2012), 637–643.

[4] D. Craven, C. Eaton, R. Kessar, M. Linckelmann, The structure of blocks with a Klein four defect group, *Math. Z.* **268** (2011), 441–476.

[5] R. Kessar, G. Malle, Quasi-isolated blocks and Brauer's height zero conjecture, submitted.

[6] M. Ascbacher, R. Kessar, B. Oliver, Fusion Systems in Algebra and Topolgy, LMS Lecture Note Series 391, Cambridge University Press **2011**.

[7] *An International Review of UK Research in Mathematics*, London Mathematical Society, 2004; available at `http://www.cms.ac.uk/irm/index.htm`.

[8] *International Review of Mathematical Sciences 2010*; available at `http://www.epsrc.ac.uk/newsevents/pubs/corporate/intrevs/2010maths`.