Advertising and price elasticity (Dorfman-Steiner result)

Demand function: $q = q(p,A)$
$C(q)$: cost function
A: advertising expenditure
Profit function:

$$\Pi = p\, q(p,A) - C[q(p,A)] - A \quad (1)$$

Two choice variables: the price p and A

$$\frac{\partial \Pi}{\partial p} = p \frac{\partial q}{\partial p} + q \frac{\partial p}{\partial p} - \frac{dC}{dq}\frac{\partial q}{\partial p} = 0 \quad (2)$$

$$\frac{\partial \Pi}{\partial A} = p \frac{\partial q}{\partial A} - \frac{dC}{dq}\frac{\partial q}{\partial A} - \frac{\partial A}{\partial A} = 0 \quad (3)$$

Take (3), multiply by A/q and rearrange:

$$p \frac{\partial q}{\partial A}\frac{A}{q} = \frac{dC}{dq}\frac{\partial q}{\partial A}\frac{A}{q} + \frac{A}{q} \quad (4)$$

note that $\frac{\partial q}{\partial A}\frac{A}{q} = \eta_A$, the elasticity of demand with respect to advertising expenditure. Also $\frac{dC}{dq} = MC$. So (4) becomes:

$$p\, \eta_A = MC\, \eta_A + \frac{A}{q} \quad (5)$$

divide both sides by p

$$\eta_A = \frac{MC}{p}\eta_A + \frac{A}{pq} \quad (6)$$

note that pq = R (revenue)

$$\frac{A}{R} = \left(\frac{p - MC}{p}\right)\eta_A \quad (7)$$

Take (2), multiply by p/q

$$p \frac{\partial q}{\partial p}\frac{p}{q} + p - \frac{dC}{dq}\frac{\partial q}{\partial p}\frac{p}{q} = 0 \quad (8)$$

Note that $-\frac{\partial q}{\partial p}\frac{p}{q} = \eta_p$, the price elasticity of demand.

$$p\,(-\eta_p) + p - MC(-\eta_p) = 0 \quad (9)$$

$$p = \eta_p(p - MC) \quad (10)$$

$$\frac{p - MC}{p} = \frac{1}{\eta_p} \quad (11)$$

Substituting (11) into (7)

$$\frac{A}{R} = \frac{\eta_A}{\eta_p}$$

So the firms's optimal level of advertising intensity (A/R) is equal to the ratio of its advertising elasticity of demand to the price elasticity of demand it faces. The important implication of this demonstration is that the level of advertising is chosen simultaneously with the level of price, there is no cause and effect relationship between these two variables.

Extending the Dorfman-Steiner Result: Conjectural Variations

For 'small numbers' oligopoly firms have to make their decisions taking into account their rivals' reactions (mutual interdependence). So the demand facing an individual firm is of the form:

$$q = q(p, A, A_r)$$

where A_r refers to the average advertising of the firm's rivals. The effect $\frac{\partial A_r}{\partial A}$ can not be neglected. It measures the extend to which I conjecture my advertising to affect that of other firms.
Profits are thus

$$\Pi = p\, q(p, A, A_r) - C[q(p, A, A_r)] - A$$

The firm maximises with respect to her own advertising, which leads to the F.O.C.

$$\frac{\partial \Pi}{\partial A} = p \frac{\partial q}{\partial A} - \frac{dC}{dq} \frac{\partial q}{\partial A} - \frac{\partial A}{\partial A} = = (p - MC) \frac{\partial q}{\partial A} - 1 = 0 \Leftrightarrow$$

$$= (p\text{-}MC) \left(\frac{\partial q}{\partial A} + \frac{\partial q}{\partial A_r} \frac{\partial A_r}{\partial A} \right) - 1 = 0 \qquad (12)$$

Multiplying the above by A/q

$$(p\text{-}MC) \left(\frac{\partial q}{\partial A} \frac{A}{q} + \frac{\partial q}{\partial A_r} \frac{A_r}{A_r} \frac{A}{q} \frac{\partial A_r}{\partial A} \right) = \frac{A}{q} \Leftrightarrow$$

$\frac{\partial q}{\partial A} \frac{A}{q} = \eta_A > 0$ elasticity of demand w.r.t. own advertising.

$\frac{\partial q}{\partial A_r} \frac{A_r}{q} = \eta_{A_r} < 0$ elasticity of demand w.r.t. the firm's rivals' advertising.

$\frac{A}{A_r} \frac{\partial A_r}{\partial A} = \eta_{A_r, A}$ elasticity of the firm's rivals advertising w.r.t.the firm's own advertising.

So (12) becomes

$$(p\text{-}MC)(\eta_A + \eta_{A_r, A}\, \eta_{A_r}) = \frac{A}{q}$$

dividing both sides by p

$$\frac{p - MC}{p} (\eta_A + \eta_{A_r, A}\, \eta_{A_r}) = \frac{A}{R}$$

Here, advertising is an **instrument of oligopolistic competition.** We expect that when η_{A_r} is negative and in absolute terms large (small numbers oligopoly), and interdependence in adverstising is high (a large $\eta_{A_r, A}$), then advertising/sales ratio will be small.