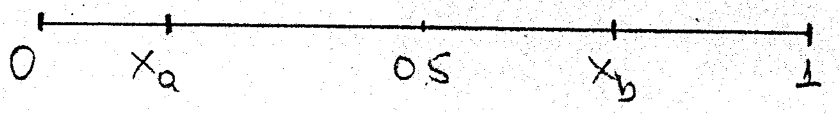
Location Competition

Linear city of length 1 and consumers are uniformly distributed. Two stores sell the same good. These two stores are located at the points x_a and x_b respectively where $x_a < x_b$ and $x_a, x_b \geq 0$. Therefore store a is to the left of store b. The unit cost of the good for each store is c. Consumers incur a transportation cost of t per unit of length.



So a consumer living at x=0 incurs a cost tx_a to go to store a and a cost tx_b to go to store b. The customer at x=0 will buy from store a as long as

$$p_a + tx_a < tx_b + p_b$$

which implies that

$$p_a - p_b < t(x_b - x_a) \quad (1)$$

So if $p_a - p_b > t(x_b - x_a)$ store b will capture the entire market and then store a will have a zero payoff.

The customer at point x=1 will buy from store b as long as

$$t(1-x_a) + p_a > t(1-x_b) + p_b$$

which implies that

$$p_b - p_a < t(x_b - x_a) \quad (2)$$

So if $p_b - p_a > t(x_b - x_a)$ store a will capture the whole of the market and have a payoff

$$\Pi_a = p_a - c$$

If inequalities (1) and (2) are both satisfied customer 0 goes to store a and customer 1 goes to store b. Let us now find the location x^* of the customer who is indifferent between the two stores. First note that if store a attracts customer x_b it also attracts all customers at $x>x_b$ because going beyond x_b customers's distances from both sellers increase at the same rate. So we know that if there is an indifferent customer at x^*, he is situated between x_a and x_b. For this customer

$$t(x^*-x_a) + p_a = t(x_b-x^*) + p_b$$

so that

$$p_b-p_a = t(2x^*-x_a-x_b)$$

Therefore,

$$x^* = \frac{1}{2t}\left((p_b-p_a)+t(x_a+x_b)\right) \tag{3}$$

Since store a keeps all customers between 0 and x^*, equation (3) is the demand curve for a so long as he does not set his price so far above b's that he looses even customer 0. The payoff of store a will then be

$$\Pi_a = (p_a-c)\ \frac{1}{2t}\left((p_b-p_a)+t(x_a+x_b)\right)$$

Store a will choose p_a so as to maximise its profits. The first order condition gives

$$\frac{\partial \Pi_a}{\partial p_a} = 0 \Leftrightarrow \frac{1}{2t}\left((p_b-p_a)+t(x_a+x_b)\right) + (p_a-c)\left(-\frac{1}{2t}\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2p_a-p_b=c+t(x_a+x_b) \tag{4}$$

The demand facing store b is $1-x^*$ and by following the same procedure we find that the first order condition for store b is

$$-p_a+2p_b = c+t(2-x_a-x_b) \tag{5}$$

Solving the two first order conditions (4) and (5) for the two prices gives:

$$p_a = c + \frac{t(2+x_a+x_b)}{3}, \quad p_b = c + \frac{t(4-x_a-x_b)}{3} \tag{6}$$

Taking into account (6), x^* in (3) is equal to

$$x^* = \frac{1}{2t}\left(c + \frac{t(4-x_a-x_b)}{3} - c - \frac{t(2+x_a+x_b)}{3} + t(x_a+x_b)\right) =$$

$$= \frac{1}{2t}\left(\frac{2t(1-x_a-x_b)}{3} + \frac{3t(x_a+x_b)}{3}\right) = \frac{1}{2t}\left(\frac{t(2+x_a+x_b)}{3}\right) = \frac{(2+x_a+x_b)}{6}$$

Consequently,

$$\Pi_a^* = (p_a-c)\,\frac{(2+x_a+x_b)}{6} = \frac{t(2+x_a+x_b)}{3}\,\frac{(2+x_a+x_b)}{6} = t\,\frac{(2+x_a+x_b)^2}{18} \tag{7}$$

Similarly we derive

$$\Pi_b^* = \frac{t(4-x_a-x_b)^2}{18} \tag{8}$$

From (7) and (8) we notice that profits are positive and increasing in the transportation cost. The products are differentiated more the higher transportation cost is. On the other hand, when $t=0$ all consumers can go to either store for the same cost (0). The absence of product differentiation leads to the Bertrand result, i.e. from (3) $p_a = p_b = c$ and from (7) and (8) $\Pi_a^* = \Pi_b^* = 0$.

Moreover, if the two firms locate at the same position the consumer will decide to which store to go purely on the basis of prices. Inequalities (1) and (2) can not be simultaneously satisfied because (1) collapses into $p_a<p_b$ and (2) into $p_b<p_a$. Therefore we have competition in terms of prices for identical products, thus leading to the Bertrand result.

Absence of price competition; the principle of minimal differentiation

Assume a city of length 1, uniform distribution of consumers, and a price p(>c), determined exogenously. Also assume that firms share demand if the are located identically. Because the prices and the profit margins are fixed, the firms choose their locations so as to maximise demand. Let firm 1 locate at point x_a and firm 2 at point x_b, where $x_a<x_b$. The demand for firm 1 is derived by setting in (3) $p_b=p_a=p$:

$$x^* = \frac{x_a+x_b}{2} \tag{9}$$

Therefore its demand increases with x_a. This is natural, as the firms compete for more consumers between them. Equally, the demand for firm 2 is:

$$1 - x^* = \frac{2-x_a-x_b}{2} \tag{10}$$

Thus its demand decreases with x_b. Consequently, an equilibrium must involve identical locations, $x_a = x_b$. Suppose now that $x_a = x_b < 1/2$. Each firm's demand is 1/2. But by moving to the right by $\varepsilon>0$, firm 2, for instance, would have demand

$$\frac{2-2x_a-\varepsilon}{2} \simeq 1-x_a > 1/2$$

Thus firms have again an incentive to move towards the centre. Therefore, when $x_a = x_b = 1/2$, neither firm would want to move. Thus, **the only equilibrium has both firms located at the centre of the city.** In this example, the products are socially close to each other. Transportation costs could be reduced by having firms move away from the center, but there is no incentive to do so.