Maths for Actuarial Science Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (C123) by **3:00pm on Thursday 12th November**. Late submissions will be penalised.

1. Solve the equation
$$\left|\frac{x-1}{x+2}\right| < \left|\frac{x+1}{x-2}\right|.$$
[10]

2. (i) Find an equation for the ellipse with foci at $(3,5)$ and $(3,7)$ and major axis of length 6.
(ii) Explain why for any circle C and point P outside it, the lengths of the two tangents from C to P are always equal. **[10]**

3. (i) Using the identities for $\cos A + \cos B$, and for $\cos^2 C$ in terms of $\cos 2C$, show that for any values of α, β, γ we have
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2(\alpha+\beta+\gamma) = 2(1+\cos(\beta+\gamma)\cos(\gamma+\alpha)\cos(\alpha+\beta)).$$
(ii) Find in terms of π the general solutions to
$$\sin 5\theta + \sin\theta = \sin 3\theta.$$
[10]

4. (i) Differentiate the following functions:
$$\text{(a)} \quad (1+3x)^2\ln(1+3x) \qquad \text{(b)} \quad \tan^2(x^3+1).$$
(ii) Find the second derivative (with respect to x) of the function
$$x = t^2 + t + 1 \qquad y = \ln(t).$$
[10]

5. Evaluate the following integrals:
$$\text{(a)} \quad \int \frac{x+2}{1-4x^2}\,dx \qquad \text{(b)} \quad \int_0^1 \frac{2x}{\sqrt{2x+1}}\,dx \qquad .$$
[10]