Maths for Actuarial Science Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the mathematics general office (C123) by 3:00pm on Thursday 12th November. Late submissions will be penalised.

1. Solve the equation \[ \left| \frac{x - 1}{x + 2} \right| < \left| \frac{x + 1}{x - 2} \right| \]. [10]

2. (i) Find an equation for the ellipse with foci at (3, 5) and (3, 7) and major axis of length 6.
(ii) Explain why for any circle C and point P outside it, the lengths of the two tangents from C to P are always equal. [10]

3. (i) Using the identities for \( \cos A + \cos B \), and for \( \cos^2 C \) in terms of \( \cos 2C \), show that for any values of \( \alpha, \beta, \gamma \) we have
\[
\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2(\alpha + \beta + \gamma) = 2(1 + \cos(\beta + \gamma) \cos(\gamma + \alpha) \cos(\alpha + \beta)).
\]
(ii) Find in terms of \( \pi \) the general solutions to
\[
\sin 5\theta + \sin \theta = \sin 3\theta.
\] [10]

4. (i) Differentiate the following functions:
(a) \((1 + 3x)^2 \ln(1 + 3x)\) \quad (b) \(\tan^2(x^3 + 1)\).
(ii) Find the second derivative (with respect to \( x \)) of the function
\[ x = t^2 + t + 1 \quad y = \ln(t). \] [10]

5. Evaluate the following integrals:
(a) \[ \int \frac{x + 2}{1 - 4x^2} \, dx \] \quad (b) \[ \int_0^1 \frac{2x}{\sqrt{2x + 1}} \, dx \]. [10]