

Mathematics for Actuarial Science: Answer sheet 2

Sheet 3

1. $x = 3, y = -\frac{1}{3}$.
2. $x = -\frac{19}{2}, y = -\frac{23}{2}, z = \frac{15}{2}$.
3. No solution.
4. $x = \frac{3-5z}{4}, y = \frac{1-7z}{4}$ (or similar).
5. $2y = 3x - 29, (5, -7)$.
6. $12y + 5x = 22, k = -22, 26$.
7. $(-4, -5), 3\sqrt{5}; (1, 0), \sqrt{5}, 5\sqrt{2}, (b) 3\sqrt{2}$.
8. $(\frac{24}{5}, \frac{32}{5})$.
9. (a) F: $(1, \frac{9}{4}), D: y = \frac{7}{4}, A: x = 1$.
 (b) F: $(-\frac{19}{4}, -1), D: x = -\frac{21}{4}, A: y = -1$.
 (c) F: $(\frac{3}{8}, \frac{1}{2}), D: y = \frac{5}{8}, A: x = \frac{3}{8}$.
10. $y^2 = 12x - 36$.
11. (a) C: $(0, 0), F: (\pm\sqrt{5}, 0),$ major: 6, minor: 4.
 (b) C: $(0, 1), F: (\pm\sqrt{5}, 1),$ major: 6, minor: 4.
 (e) C: $(0, 3), F: (0, 3 \pm \sqrt{3}),$ major: 4, minor: 2.
12. (a) $\frac{x^2}{9} + \frac{y^2}{8} = 1$.
 (b) $\frac{(x-1)^2}{21} + \frac{(y-3)^2}{25} = 1$.
 (c) $\frac{x^2}{25} + \frac{(y+1)^2}{9} = 1$.
13. (a) C: $(0, 0), F: (\pm\sqrt{2}, 0), A: y = \pm x$.
 (b) C: $(0, 0), F: (\pm 5, 0), A: y = \pm \frac{4}{3}x$.
 (c) C: $(1, 3), F: (1, 3 \pm \sqrt{5}),$
 A: $y = 2x + 1$ and $y = -2x + 5$.

Sheet 4

1. (a) $12x^2 - 4x - 2x^{-3}$.
 (b) $-3 \sin(3x + 2)$.
 (c) $\frac{1-x^2}{(1+x^2)^2}$.
 (d) $1 + \ln x$.
 (e) $\frac{x}{\sqrt{1+x^2}}$.
 (f) $e^x \cos(x^2) - 2xe^x \sin(x^2)$.
 (g) $\frac{6}{(2x-1)^2} - \frac{2}{(x+2)^2}$.
 (h) $x^{e^x} (\frac{1}{x}e^x + \ln(x)e^x)$.
 (i) $-\sin(\tan(x^2)) \sec^2(x^2)2x$.
2. (a) $3x^2 + 3y^2 + 6xy \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} - 4x^{-5}y^{-3} - 3x^{-4}y^{-4} \frac{dy}{dx} = 0$.
 (b) $-\sin(x) \sin(y) + \cos(x) \cos(y) \frac{dy}{dx} = 0$.
 (c) $-\sin(y \tan(x)) (\frac{dy}{dx} \tan(x) + y \sec^2(x)) = (xy)^{-1} (y + x \frac{dy}{dx})$.
 (d) $\frac{(1+2 \cos(y) \frac{dy}{dx})(x-y) - (x+2 \sin(y)+4)(1-\frac{dy}{dx})}{(x-y)^2} = -\operatorname{cosec}^2(y) \frac{dy}{dx}$.
 (e) $\tan(x^{-1} \frac{dy}{dx} - y \sec^2(x^{-1})x^{-2} + \sec(y^{-1}) - x \sec(y^{-1}) \tan(y^{-1})y^{-2} \frac{dy}{dx} = e^x$.
3. (a) $\frac{21t^2}{2t+2}$
 (b) $\operatorname{cosec}^3(t)$
 (c) $\frac{(3t^2+1)(1+2t^2)}{2t}$
 (d) $e^{-2t^2} \frac{\cos(t)-2t \sin(t)}{2t \cos(t)-\sin(t)}$
 (e) $-12t \sec^2(4t) \ln(t)^2$.
4. 8.
5. $f'(x) = -\frac{x}{2} + \frac{1}{x}, \beta = \sqrt{2}$.