

Mathematics for Actuarial Science: Answer sheet 5

Sheet 9

1. (a) (i) 1, 2, 3. (ii) $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.
(b) (i) $R, \{G\}, B$. (ii) $\emptyset, \{R\}, \{\{G\}\}, \{B\}, \{R, \{G\}\}, \{R, B\}, \{\{G\}, B\}, \{R, \{G\}, B\}$.
(c) (i) $R, \{R\}$. (ii) $\emptyset, \{R\}, \{\{R\}\}, \{R, \{R\}\}$.
2. (a) T , (b) F , (c) F , (d) T , (e) T , (f) F .
3. (a) F , e.g. $A = \{1\}, B = \{2\}, C = \{1, 2\}$, (b) T , (c) F , could have $A = B = C$, (d) F , could have $A = B$, (e) T .
5. Various possible examples. It is best to split the sentence into the simplest components. We will give sample solutions.
 - (a) Let $m =$ “I will go out for a meal”, $f =$ “I will see a film”, $c =$ “I finish my coursework”, and $r =$ “run out of money”. Then the sentence becomes
$$(c \wedge (\neg r)) \longrightarrow (m \wedge f).$$
 - (b) Let $r =$ “I will cut the red wire”, $g =$ “I will cut the green wire”, $b =$ “I will cut the blue wire”, $e =$ “the bomb will explode”, and $s =$ “we will be saved”. Then the sentence becomes
$$(r \vee (g \wedge b)) \longrightarrow ((\neg e) \wedge s).$$
6. (a) T , (b) F , (c) F , (d) T , (e) T , (f) F , (g) T .
7. False when $p = F, q = F, r = F$.
9.
$$\neg (\{[(\exists x)(\forall t)p(x, t)] \wedge [(\forall x)(\exists t)p(x, t)]\} \longrightarrow [(\forall x)(\forall t)p(x, t)])$$
10. (a) F , (b) F , (c) T , (d) T , (e) F .

Sheet 10

As they are proofs, answers for Sheet 10 are not given here.