

## Mathematics for Actuarial Science 10

1. Let  $n$  and  $m$  be positive integers. We say that  $m$  *divides*  $n$ , denoted  $m|n$ , if there exists a positive integer  $k$  with  $n = km$ . Prove
  - (a) If  $m|n$  and  $n|p$  then  $m|p$ .
  - (b) If  $m|n$  and  $n|m$  then  $m = n$ .
  - (c) If  $p|m$  and  $p|n$  then  $p|(am + bn)$  for all positive integers  $a$  and  $b$ .
2. Give a contrapositive proof that if  $n$  is a perfect number then  $n$  is not a prime number. (A *perfect* number is equal to the sum of its factors: e.g.  $6 = 1 + 2 + 3$ .)
3. Modify the proof of the irrationality of  $\sqrt{2}$  given in lectures to show that  $\sqrt{3}$  is irrational. Why does a similar proof not show that  $\sqrt{4}$  is irrational?

4. Prove by induction that

$$\sum_{i=1}^n i^2 = \frac{1}{6}n(n+1)(2n+1).$$

5. Prove by induction that

$$\sum_{r=1}^n \frac{1}{r(r+1)(r+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$

6. Prove by induction that  $2^{n+2} + 3^{2n+1}$  is divisible by 7.