

### Mathematics for Actuarial Science 5

1. Find the gradient of the curve at the point for which  $x = 4$  in each of the following cases:

(a)  $y = x^2 - 3\sqrt{x}$ ,

(b)  $y = \cos(\frac{1}{4}\pi x)$ .

2. Find the equation of the tangent at the point  $(1, 0)$  to the curve  $y = (x + 1) \ln x$ ,  $x > 0$ , and verify that it meets the line  $x = 2$  at the point  $(2, 2)$ .

3. Given  $y = e^{-x^2}$  find expressions for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . Hence find the  $x$  coordinates of the two points on the graph for which  $\frac{d^2y}{dx^2}$  is equal to zero. Show that these are both points of inflection.

4. Find the maximum and minimum values of  $4 \sin x + \frac{9}{(1 + \sin x)}$  for  $0 \leq x \leq \pi$ .

5. The function  $f$  is defined by  $f(x) = 4x^2 - 3 - \frac{1}{x}$  with  $x \neq 0$ .

(a) Find  $f'(x)$  and  $f''(x)$ .

(b) Find the values of  $a$  such that  $f'(a) = 0$ , and calculate  $f''(a)$  in these cases. What information does this give you about the graph of  $f$ ?

(c) Find the values of  $x$  where the graph  $y = f(x)$  meets the  $x$ -axis.

(d) Sketch the curve  $y = f(x)$ . Explain how the shape of the graph is related to the terms in the expression for  $f(x)$ : (i) when  $x$  is near 0 and (ii) when  $x$  is large (positive or negative).

6. Express  $f(x) = \frac{(3x^2 + 1)^2}{x^2}$  in the form  $Ax^2 + Bx + \frac{C}{x^2}$ . Hence evaluate

$$\int_1^2 f(x) dx.$$

7. Differentiate  $\ln(x^2 - 2x + 2)$  with respect to  $x$ . Hence find

$$\int_1^2 \frac{x - 1}{x^2 - 2x + 2} dx.$$

8. Evaluate the integral

$$\int_0^1 \frac{x^3}{1 + x^4} dx.$$

9. Using the substitution  $y = 2x - 1$ , evaluate the integral

$$\int_1^2 \frac{x}{(2x - 1)^3} dx.$$

10. Express  $f(x) = \frac{4 - 6x^2}{(1 + 2x)^2(1 + x)}$  in partial fractions. Show that

$$\int_0^1 f(x) dx = \frac{5}{3} + \ln \frac{\sqrt{3}}{4}.$$

11. Find

$$\int \sin^2 x \cos x dx.$$

12. Find

$$\int \frac{e^{3x} - e^{2x}}{e^x} dx.$$

13. Find

$$\int x\sqrt{4x - 3} dx.$$

14. Find

$$\int \tan x \sec^2 x dx.$$

15. Find

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx.$$

16. Find

$$\int (x - 5)^4 (x + 3)^2 dx.$$

17. Find

$$\int \frac{\sin 3x}{1 + \cos 3x} dx.$$

18. Find

$$\int 2 \sin 7x \cos 3x dx.$$

19. Find

$$\int \cos^4 x dx.$$