1. Find the gradient of the curve at the point for which \( x = 4 \) in each of the following cases:
   (a) \( y = x^2 - 3\sqrt{x} \).
   (b) \( y = \cos(\frac{x}{2}) \).
2. Find the equation of the tangent at the point \((1, 0)\) to the curve \( y = (x + 1)\ln x \), \( x > 0 \), and verify that it meets the line \( x = 2 \) at the point \((2, 2)\).
3. Given \( y = e^{-x^2} \) find expressions for \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \). Hence find the \( x \)-coordinates of the two points on the graph for which \( \frac{d^2y}{dx^2} \) is equal to zero. Show that these are both points of inflection.
4. Find the maximum and minimum values of \( 4\sin x + \frac{9}{(1+\sin x)^2} \) for \( 0 \leq x \leq \pi \).
5. The function \( f \) is defined by \( f(x) = 4x^2 - 3 - \frac{1}{x} \) with \( x \neq 0 \).
   (a) Find \( f'(x) \) and \( f''(x) \).
   (b) Find the values of \( a \) such that \( f'(a) = 0 \), and calculate \( f''(a) \) in these cases. What information does this give you about the graph of \( f' \)?
   (c) Find the values of \( x \) where the graph \( y = f(x) \) meets the \( x \)-axis.
   (d) Sketch the curve \( y = f(x) \). Explain how the shape of the graph is related to the terms in the expression for \( f(x) \): (i) when \( x \) is near 0 and (ii) when \( x \) is large (positive or negative).
6. Express \( f(x) = \frac{4x^4 + x^2}{x^2} \) in the form \( Ax^2 + Bx + C \) \( x \neq 0 \). Hence evaluate \( \int_1^2 f(x)\,dx \).
7. Differentiate \( \ln(x^2 - 2x + 2) \) with respect to \( x \). Hence find \( \int_1^2 \frac{x - 1}{x^2 - 2x + 2}\,dx \).
8. Evaluate the integral \( \int_0^1 \frac{x^3}{1 + x^4}\,dx \).
9. Using the substitution \( y = 2x - 1 \), evaluate the integral \( \int_1^2 \frac{x}{(2x - 1)^3}\,dx \).
10. Express \( f(x) = \frac{4\ln^4 x}{(x^2 + 2\ln x)} \) in partial fractions. Show that \( \int_0^1 f(x)\,dx = \frac{5}{3} + \ln\frac{\sqrt{3}}{4} \).
11. Find \( \int \sin^2 x \cos x\,dx \).
12. Find \( \int e^{2x} - e^{3x}\,dx \).
13. Find \( \int x\sqrt{4x - 3}\,dx \).
14. Find \( \int \tan x \sec^2 x\,dx \).
15. Find \( \int \sin \sqrt{x}\,dx \).
16. Find \( \int (x - 5)^4(x + 3)^2\,dx \).
17. Find \( \int \frac{\sin 3x}{1 + \cos 3x}\,dx \).
18. Find \( \int 2\sin 7x \cos 3x\,dx \).
19. Find \( \int \cos^3 x\,dx \).