Mathematics for Actuarial Science 6

1. Calculate
$$\text{(a)} \int x\ln x\,dx \qquad \text{(b)} \int x(\ln x)^2 dx.$$

2. Calculate
$$\text{(a)} \int \sqrt{3x+8}dx \qquad \text{(b)} \int (3x+8)e^x dx \qquad \text{(c)} \int 3x\sqrt{3x+8}dx.$$

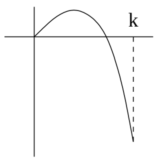
3. (a) Given that $2y = x - \sin x\cos x$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin^2 x$.
   (b) Hence find
$$\int x\sin^2 x dx.$$

4. Calculate
$$\int_0^\pi x^2\cos 3x dx.$$

5. Let I_n stand for the integral $\int x^n e^{2x}dx$. Use integration by parts to give a formula relating I_n to I_{n-1}. Use this result to find I_4.

6. The curve with equation $y = e^{3x} + 1$ meets the line $y = 8$ at the point $(h, 8)$.
   (a) Find h, giving your answer in terms of natural logarithms.
   (b) Show that the area of the finite region enclosed by the curve with equation $y = e^{3x} + 1$, the x-axis, the y-axis, and the line $x = h$, is $2 + \frac{1}{3}\ln 7$.

7. The graph of $y = x(4 - x^2)$ is illustrated below for $x \geq 0$. Find the exact value of k for which the areas above and below the x-axis are equal.



8. Curves C and D have equations $y = \frac{1}{x}$ and $y = kx^2$ respectively, where k is a constant. The curves intersect at the point P, whose x-coordinate is $\frac{1}{2}$.
   (a) Determine the value of k.
   (b) Find the gradient of C at P.
   (c) Calculate the area of the finite region bounded by C, D, the x-axis, and the line $x = 2$.

9. Simplify $\tan(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4})$.

10. Solve the equation
$$\sin^{-1}\left(\frac{x}{x-1}\right) + 2\tan^{-1}\left(\frac{1}{x+1}\right) = \frac{\pi}{2}.$$

11. Calculate
$$\int \frac{3}{2x^2+5}dx.$$