

Mathematics for Actuarial Science 8

1. Determine the following limits:

(a)

$$\lim_{x \rightarrow -2} \frac{x^2 + 6x + 8}{x + 2}.$$

(b)

$$\lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x^2 - 4}}.$$

(c)

$$\lim_{x \rightarrow 0} \frac{\tan x}{x}.$$

(d)

$$\lim_{x \rightarrow 0} \frac{1 - \cos(ax)}{x^2}.$$

(e)

$$\lim_{x \rightarrow 0} \frac{(x^2 + 2) \sin(3x)}{x}.$$

(f)

$$\lim_{x \rightarrow \infty} \frac{x^2 + x + 1}{x^2 - x + 1}.$$

(g)

$$\lim_{x \rightarrow 0} \frac{\tanh x}{x}.$$

(h)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x + \sqrt{x + 1}}}{\sqrt{x + \sqrt{1 + \sqrt{x}}}}.$$

2. Give the Maclaurin series for each of the following functions:

(a) $2 \sin x \cos x$.

(b) $\ln \sqrt{\frac{1-x}{1+x}}$.

(c) $\ln[(1+x)(1-2x)^2]$.

(d) $\sin^3 x$.

3. Give the Taylor series for $\cos x$ about $\frac{\pi}{2}$ to three terms.

4. If $y = \tan(e^x - 1)$, prove that

$$\frac{d^2 y}{dx^2} = \frac{dy}{dx} (1 + 2e^x y).$$

Obtain the Maclaurin expansion of y up to and including the x^4 term.

5. Use Leibnitz' theorem to compute the fifth derivative of

$$(a) \ x^3 \cos x \quad (b) \ x^2 \ln x.$$

6. If $y = \frac{x+1}{x^2+2x+3}$ show that

$$(x^2 + 2x + 3) \frac{d^2 y}{dx^2} + 4(x + 1) \frac{dy}{dx} + 2y = 0.$$

Use Leibnitz' theorem to establish that

$$(x^2 + 2x + 3) \frac{d^{n+2} y}{dx^{n+2}} + 2(n + 2)(x + 1) \frac{d^{n+1} y}{dx^{n+1}} + (n + 1)(n + 2) \frac{d^n y}{dx^n} = 0.$$