Mathematics for Actuarial Science 9

1. For each of the following sets determine (i) the elements of the set and (ii) the subsets of the set.

   (a) $A = \{1, 2, 3\}$

   (b) $B = \{R, \{G\}, B\}$

   (c) $C = \{R, \{R\}\}$

2. Determine whether each of the following statements is true or false.

   (a) $2 \in \{1, 2, 3\}$.

   (b) $2 \subset \{1, 2, 3\}$.

   (c) $\{2\} \in \{1, 2, 3\}$.

   (d) $\{2\} \subseteq \{1, 2, 3\}$.

   (e) $\emptyset \in \{\emptyset\}$.

   (f) $\emptyset = \{\emptyset\}$.

3. For each of the following statements, determine (with reasons) whether it is true or false. For each false statement, give an example to show that it fails.

   (a) If $A \subset C$ and $B \subseteq C$ then $A \subset B$.

   (b) If $A \subset B$ and $B \subset C$ then $A \subseteq C$.

   (c) If $A \subseteq B$ and $B \subseteq C$ then $A \subset C$.

   (d) If $A \subseteq B$ and $A \cap B \neq \emptyset$ then $A \subset B$.

   (e) If $A \subseteq B$ and $A \cup B \neq A$ then $A \subset B$.

4. Illustrate the following identities using Venn diagrams, then prove them using membership tables.

   (a) $(A \cap B) \cap C = A \cap (B \cap C)$.

   (b) $A \backslash (B \cup C) = (A \backslash C) \cap (A \backslash B)$.

5. Symbolise each of the following propositions.

   (a) I will go out for a meal and see a film if I both finish my coursework and do not run out of money.

   (b) If I cut the red wire, or cut both the green wire and the blue wire, then the bomb will not explode and we shall be saved.

6. Let p be the proposition "$2 + 3 = 5$", q be the proposition "Paris is the capital of Germany" and r be the proposition "some bananas are yellow". Determine whether each of the following is true or false.

   (a) $p \vee q$.

   (b) $r \wedge q$.

   (c) $(\neg r) \vee q$.

   (d) $\neg(q \vee (\neg r))$.

   (e) $q \rightarrow r$.

   (f) $(\neg q) \rightarrow (\neg p)$.

   (g) $\neg(r \rightarrow q)$.

7. By using truth tables, determine when the proposition $p \vee ((\neg q) \rightarrow r)$ can be false, in terms of the truth values for p, q and r.

8. Use truth tables to show that $(p \rightarrow q) \wedge r$ is not logically equivalent to $p \rightarrow (q \wedge r)$.

9. Let $p(x, t)$ be the predicate "you can fool x at time t" where x comes from the set of people and t comes from the set of moments in time. Symbolise the following:

   > If you can fool some of the people all of the time, and all of the people some of the time, it does not meean you can fool all of the people all of the time.

10. Let $p(x)$ be the predicate "$x > 0$" and $n(x)$ be the predicate "$x < 0$" where $x \in \mathbb{Z}$. Determine whether each of the following is true or false.

    (a) $(\forall x)(p(x) \vee (\neg n(x)))$.

    (b) $(\exists x)(p(x) \wedge n(x))$.

    (c) $((\exists x)p(x)) \wedge ((\exists x)\neg n(x))$.

    (d) $(\exists x)(\exists y)(p(x) \wedge n(y))$.

    (e) $((\forall x)p(x)) \vee (\neg((\exists x)n(x)))$.