Linear Algebra 2

1. Check that the set of vectors $\{\mathbf{e_1} = (1,0,0), \mathbf{e_2} = (0,1,0), \mathbf{f} = (1,1,1)\}$ in Example 1.16 is linearly independent in $\mathbb{R}^3$.

2. Check (Example 1.18) that the above set of vectors form a basis of $\mathbb{R}^3$, but that the set $\{\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}, \mathbf{f}\}$ (where $\mathbf{e_3} = (0,0,1)$) does not.

3. Verify that the standard bases given in Examples 1.18, 1.22 and 1.23, for $\mathbb{R}^n$, P_n and $M(m,n)$ respectively, are bases over $\mathbb{R}$.

4. Let $\mathbf{e_i} = (0,\ldots,0,1,0,\ldots,0) \in \mathbb{C}^n$ be the vector with a 1 in the ith entry and zeros elsewhere.

   (a) Show that $\{\mathbf{e_1},\ldots,\mathbf{e_n}\}$ is a basis for $\mathbb{C}^n$ as a vector space over $\mathbb{C}$.

   (b) Show that $\{\mathbf{e_1},\ldots,\mathbf{e_n}\}$ is *not* a basis for $\mathbb{C}^n$ as a vector space over $\mathbb{R}$.

   (c) Show that $\{\mathbf{e_1},\ldots,\mathbf{e_n}, i\mathbf{e_1},\ldots,i\mathbf{e_n}\}$ is a basis for $\mathbb{C}^n$ as a vector space over $\mathbb{R}$.

5. Which of the following sets are bases for $\mathbb{R}^3$?

   (a) $\{(1,0,0),(2,2,0),(3,3,3)\}$.

   (b) $\{(2,1,3),(6,3,9),(3,1,2)\}$.

   (c) $\{(3,1,-4),(2,5,6),(5,6,2)\}$.

   (d) $\{(1,6,4),(2,4,-1),(-1,2,5)\}$.

6. Which of the following sets are bases for P_2?

   (a) $\{1+x+x^2, x+x^2, x^2\}$.

   (b) $\{1-3x+2x^2, 1+x+4x^2, 1-7x\}$.

   (c) $\{-1+x+2x^2, 1-x+x^2, 3x^2\}$.

   (d) $\{-4+x+3x^2, 6+5x+2x^2, 8+4x+x^2\}$.

7. Let $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be a basis for a vector space V. Show that $\{\mathbf{v_1}, \mathbf{v_2}+\mathbf{v_1}, \mathbf{v_3}+\mathbf{v_2}+\mathbf{v_1}\}$ is also a basis for V.

8. (Optional) Show that the space of all functions $f: \mathbb{R} \to \mathbb{R}$ is not a finite dimensional vector space over $\mathbb{R}$. (Hint: use polynomials.)

9. (Optional) Complete the proof of Corollary 1.33.