Linear Algebra 4

1. Write each of the following vectors in coordinate form with respect to the given basis.

   (a) Vector: $3x^2 + 2x + 5$; Basis in P_2: $x^2 + 1,\ x - 3,\ x + 1$.

   (b) Vector: $\begin{pmatrix} 1 & 3 \\ 7 & 2 \end{pmatrix}$; Basis in $M(2,2)$: $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

   (c) Vector: $a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$; Basis (inside the vector space spanned by $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$): $\mathbf{u}_1, \mathbf{u}_2 + \mathbf{u}_1, \mathbf{u}_3 + \mathbf{u}_2 + \mathbf{u}_1$

2. For each of the following maps $f : U \to V$, determine (as in Example 2.10) the matrix for f with respect to the given bases of U and V.

   (a) Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be the map given on the standard basis by $f(\mathbf{e}_1) = 3\mathbf{e}_1 + \mathbf{e}_2 + 2\mathbf{e}_3$ and $f(\mathbf{e}_2) = \mathbf{e}_1 + 5\mathbf{e}_2 + 4\mathbf{e}_3$. Take as the basis of $\mathbb{R}^2$ the elements $\mathbf{e}_1 + 2\mathbf{e}_2$ and $2\mathbf{e}_2$, and take the standard basis for $\mathbb{R}^3$.

   (b) Let f, U and V be as in part (a), but now take the standard basis for $\mathbb{R}^2$ and the basis for $\mathbb{R}^3$ given by the elements $\mathbf{e}_1 + 4\mathbf{e}_2$, $\mathbf{e}_2 + 2\mathbf{e}_3$, $\mathbf{e}_1$.

   (c) Let $f : P_2 \to P_2$ be the map $f(p) = \frac{d}{dx}(p)$. Take as the basis both of $U = P_2$ and of $V = P_2$ the standard basis.

   (d) Let f, U and V be as in part (c), with the standard basis for $U = P_2$, but now take the basis for $V = P_2$ given by the elements $x^2 + 1,\ x - 3,\ x + 1$.

   (e) Let f, U and V be as in part (c), with the basis for $U = P_2$ given by the elements $x + 3,\ x^2 + x + 1,\ x^2 - 1$, and the basis for $V = P_2$ given by the elements $x^2 + 1$, $x - 3,\ x + 1$.