Linear Algebra 6

1. Verify that the Cayley-Hamilton theorem holds for the matrix

$$\begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}.$$

2. For each of the following functions, either prove that they give a real inner product, or give an example to show how they fail.

   (a) $\langle -, - \rangle$ on $\mathbb{R}^3$ given by $\langle \mathbf{x}, \mathbf{y} \rangle = 2x_1y_1 + 4x_2y_2 + 6x_3y_3$.

   (b) $\langle -, - \rangle$ on $\mathbb{R}^3$ given by $\langle \mathbf{x}, \mathbf{y} \rangle = 2x_1y_1 + 4x_2y_2 - 6x_3y_3$.

   (c) $\langle -, - \rangle$ on $\mathbb{R}^3$ given by $\langle \mathbf{x}, \mathbf{y} \rangle = x_1y_2 + x_2y_3 + x_3y_1$.

   (d) $\langle -, - \rangle$ on P_2 given by $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$.

   (e) $\langle -, - \rangle$ on P_2 given by $\langle p, q \rangle = p_0q_0 + p_1q_1 + p_2q_2$, where $p(x) = p_0 + p_1x + p_2x^2$ and $q(x) = q_0 + q_1x + q_2x^2$.

   (f) $\langle -, - \rangle$ on P_2 given by $\langle p, q \rangle = p_0q_0 + p_1q_1$, where $p(x) = p_0 + p_1x + p_2x^2$ and $q(x) = q_0 + q_1x + q_2x^2$.

   (g) $\langle -, - \rangle$ on $M(2, 2)$ given by $\langle A, B \rangle = \mathrm{tr}(A + B)$.

3. Calculate the inner product $\langle A, B \rangle$ of $A = \begin{pmatrix} 1 & 3 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -4 \\ 0 & -1 \end{pmatrix}$ using the inner product given on matrices in the lectures (Example 4.4).

4. Calculate the inner product $\langle p, q \rangle$ of $p = 1 + x^2$ and $q = 2 - 3x$ for each of the functions 2(d)-(f) which is an inner product.