Revision notes for Linear Algebra

There will not be any revision lectures for Linear Algebra this year, as I will be away from the University. These notes are intended to give some guidance as to suitable topics to revise, and to remind you of the the main points in the course.

Past exams

I have set the exam in Linear algebra since 2002, and any past questions from those years would be suitable revision practice. From earlier papers you might look at

2001: Q5 b,c,e and Q6.
2000: Q6 a and Q7.
1999: Q5 and Q8.
1998: Q5(?), Q6 and Q8.
1997: Q5 a, Q6 a,b, Q7 (and Q8?).

You should also make sure you have revised any questions which have arisen on the courseworks, or on the examples sheets. It is important to note that you will be expected to state important definitions and (some of the more important results) in the exam, but you will not be expected to reproduce the detailed proofs which I gave.

Course content

I list the most important topics only (I may have forgotten something, but hope that this is pretty exhaustive...):

Chapter 1: Definition of a vector space, and standard examples. Definition of a subspace. Definition of linear combination, span, and spanning set. Definition of linear independence and basis. Uniqueness of the expression for a general vector as a sum of basis elements. The standard ordered bases for our main examples. Definition of a finite dimensional vector space. General results about bases including: size of a linearly independent set is less than or equal to the size of a spanning set, all bases have the same number of elements. Definition of dimension, relation between dimensions of a subspace and the whole space, conditions for a set of n vectors to be a basis in an n-dimensional space.

Chapter 2: Definition of a linear map, and of injective/surjective/isomorphism. Definition of image and kernel, and that they are subspaces. The rank-nullity theorem. How to get a linear map from a matrix, and vice versa. Definition of row span and row rank.

Chapter 3: Definition of eigenvectors/spaces/values, and how to calculate them. The diagonalisation theorem and the special case of n distinct eigenvalues. Definition of similar matrices. The characteristic equation and the Cayley-Hamilton Theorem.

Chapter 4: Definition of a real inner product, of a norm, and of a real inner product space; how to verify the standard examples. Cauchy-Schwartz inequality. Orthogonal and orthonormal vectors, and the expression for a general vector as a sum of orthonormal basis elements. Orthogonal vectors are linearly independent. How to construct an orthonormal basis (using Gram-Schmidt). Orthogonal and symmetric matrices, and their properties. The orthogonal diagonalisation theorem.