Mathematical communication 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the general office (C123) by **3:00pm on Tuesday 13th November**. Late submissions will be penalised.

1. (a) Let x and y be arbitrary real numbers. For each of the following examples state whether the given condition is necessary, sufficient, both, or neither. For those which are not necessary give an example to demonstrate this, and similarly for those which are not sufficient.

   i. Condition: $x \geq y$; conclusion: $(x-y)^2 > 0$.

   ii. Condition: $x^2 + y^2 - 2xy > 0$; conclusion: $x < y$.

   iii. Condition: $x = -y$; conclusion: $x^2 + y^2 + 2xy = 0$.

   iv. Condition: $-1 < x < y < 1$; conclusion: $xy < 1$.

   v. Condition: $xy < 1$; conclusion: $-1 < x < y < 1$.

   (b) Explain the relation between your answers to parts (iv) and (v) above.

[15]

2. Read Chapter III (covering rules 9 to 18) of Strunk's "The Elements of Style". In light of the rules of usage given there, find five ways in which the following passage is poorly written, explain what they are, and illustrate them by rewriting it appropriately.

> An examination was sat by me yesterday. The paper was not without some difficult questions! In spite of the fact that I had prepared very well, I struggled towards the end: on the first page I completed most of the questions while on the second side I finished only two. I hope, despite my problems with some of the later questions, that I have passed.

[15]

3. Let f be a quadratic polynomial such that the value of f at one is three times the value of f at minus one. Then the value of f at zero plus the value of f at minus four depends only on the second derivative of f.

   Express this result in symbols and hence explain why it is true.

[15]

4. You are given a list of real numbers, and told to perform the following operations. For each number on the list apart from the first and the last, multiply the number by what remains when the number before it in the list is subtracted from the number that follows it. (That is, multiply the number by the difference between its two neighbours.) If the first and last numbers are both zero then the sum of the numbers obtained will also be zero.

   Express this result in symbols and hence explain why it is true.

[15]

5. (a) Let $A = \{0, \{1, 2\}, 3\}$. Make a list of the elements of A and a (separate) list of the subsets.

   (b) In each of the following cases determine whether $A \subset B$, $A = B$, or $A \not\subseteq B$. Give reasons for your answers.

   i. A is the set of all integers, B is the set of all real numbers x such that $p(x) = 0$ for some quadratic polynomial p.

   ii. A is the set of all integer multiples of 2, B is the set of all integer multiples of -2.

   iii. $A = \{\emptyset, 1, 2\}$ and $B = \{\emptyset, 1, \{2\}, \{\emptyset\}\}$ (recall that $\emptyset$ denotes the empty set).

[15]

6. For each of the following statements, determine (with reasons) whether it is true or false. For each false statement, give an example to show that it fails.

   (a) If $A \subset B$ and $B \subset C$ then $A \subseteq C$.

   (b) If $A \subset B$ and $B \subseteq C$ then $A \subset C$.

   (c) If $A \subseteq B$ and $A \subset C$ then $B \subseteq C$.

   (d) If $A \subseteq B$ and $A \cap (B') \neq \emptyset$ then $A \subset B$.

[15]

7. Let A, B and C be sets inside our universal set U. Illustrate the following sets using Venn diagrams. (Note that your Venn diagrams should contain precisely eight different regions.)

   (a) $(A \cup (C')) \cap B$.

   (b) $(A \cap B) \cup C$.

   (c) $((A')\backslash(B')) \cup (C\backslash B)$.

[10]