Mathematical communication 2

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the general office (C123) by **3:00pm on Friday 18th January**. Late submissions will be penalised.

1. Denote by A the set of square numbers, B the set of natural numbers divisible by 3, C the set of natural numbers less than 50 and D the set $\{3, 4, 7, 15, 20, 30, 81, 101\}$.

   (a) Identify the following sets

   i. $(C' \cap D)\backslash D$.

   ii. $((A \cap C) \cap B)\backslash D$.

   (b) Write the following sets in terms of A, B, C, and D, using $\cup$, $\cap$, $'$ and $\backslash$.

   i. $\{4, 81, 101\}$.

   ii. $\{4, 7, 20\}$.

   iii. $\emptyset$.

   [10]

2. For each of the following maps f, determine whether it is injective, surjective, both, or neither. For each non-injective map, give an example of the failure of injectivity, and similarly for the non-surjective maps.

   (a) $f : \{a, b, c, d\} \to \{1, 3, 5, 7\}$ given by $a \mapsto 3$, $b \mapsto 5$, $c \mapsto 1$, and $d \mapsto 5$.

   (b) $f : \{1, 2, 3\} \to \{2, 4, 6, 8\}$ given by $f(x) = 2x$.

   (c) $f : \mathbb{Z} \to \mathbb{N}$ given by $f(x) = x^2 + x + 1$.

   (d) $f : \mathbb{N} \to \mathbb{Z}$ given by $f(z) = 7 - z$.

   (e) Given two sets A and B, and a bijection $g : B \to A$, the map $f : A \to B$ given by $f(x) = y$ where $g(y) = x$. (In other words f is the *inverse map* to g.)

   [15]

3. Show that the set of all cubes $\{\ldots, -27, -8, -1, 0, 1, 8, 27, \ldots\}$ has cardinality $\aleph_0$.

   [10]

4. Prove (using the method in the notes — not membership tables!) that for all sets A, B, and C we have the identity

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C).$$

   [15]

5. (a) For each of the following statements, determine whether it is a proposition. If it is a proposition, state whether it is true or false.

   i. Either this is not a proposition or its negation is false.

   ii. Tell me if this a proposition

   iii. Is this a true proposition?

   (b) Let p be the proposition "$2 + 2 = 5$", q be the proposition "the earth is not flat" and r be the proposition "3 is an odd number". Determine whether each of the following is true or false.

   i. $\neg(p \vee q)$.

   ii. $(\neg q) \to (\neg r)$.

   iii. $\neg(q \to r)$.

   iv. $(p \vee r) \to q$.

   v. $(\neg q) \to (p \wedge r)$.

   vi. $((q \wedge r) \to p) \to p$. **[15]**

6. Symbolise each of the following propositions.

   (a) I will go out for a meal and see a film if I both finish my coursework and do not run out of money.

   (b) If I cut the red wire, or cut both the green wire and the blue wire, then the bomb will not explode and we shall be saved.

   [10]

7. Use truth tables to show that $(p \to q) \wedge r$ is not logically equivalent to $p \to (q \wedge r)$.

   [10]

8. In the following list, find three statements that are either ambiguous or misuse logical symbols. The remaining four fall into two pairs of logically equivalent propositions, which you should identify.

   (a) $\neg p \to (q \to r)$.

   (b) $(r \wedge p) \to (p \vee q)$.

   (c) $r \to ((\neg p) \vee q)$.

   (d) $((q \wedge p)q) \to q$.

   (e) $((\neg p) \vee (\neg q) \to r$.

   (f) $(p \wedge (\neg q)) \to (\neg r)$.

   (g) $(p \vee q) \to (r \vee (\neg r))$.

   [15]