Mathematical Communication Mock Exam Answers

The following solutions are not complete, as they do not contain full working. They are intended to provide a quick indication as to whether you have solved the questions correctly. In an exam you would need to give full details.

Section A

Question 1:

A necessary condition is one that must hold for the given conclusion to occur. A sufficient condition guarantees that the given conclusion will occur.

(a) Neither. Not necessary, eg $x = 1$ and $y = -4$.
Not sufficient, eg $x = -1$ and $y = 4$.

(b) The condition is sufficient as it rearranges to give $y > 0$. Not necessary, eg $x = 1$ and $y = 0$.

(c) Sufficient, as can divide both sides by x^2. Not necessary, eg $x = 0$ and $y = 1$.

Question 2:

(a) FALSE, eg $A = B = C$.

(b) FALSE, eg $A = \{1\}$, $B = \{1, 2, 3\}$ and $C = \{3\}$.

(c) FALSE, eg $A = U$ (the universe) and C is a non-empty proper subset of U.

(d) TRUE, as $B \backslash A \neq \emptyset$ is contained in $A \cup B$ but not in $A \cap B$.

Question 3:

The truth table is given by

p	q	r	$q \to r$	$p \vee (q \to r)$	$p \vee q$	$(p \vee q) \to r$
T	T	T	T	T	T	T
T	T	F	F	T	T	F
T	F	T	T	T	T	T
T	F	F	T	T	T	F
F	T	T	T	T	T	T
F	T	F	F	F	T	F
F	F	T	T	T	F	T
F	F	F	T	T	F	T

The columns for $p \vee (q \to r)$ and $(p \vee q) \to r)$ are different so the propositions are inequivalent.

Question 4:

(a) The contrapositive of $p \to q$ is $(\neg q) \to (\neg p)$.

(b) A contradiction is a proposition which is never true, e.g. $p \wedge (\neg p)$.

(c) $p = T$, $q = F$, $r = F$, so the answers are
(i) misuse of symbols as s is not defined, (ii) misuse of symbols as brackets missing, (iii) True, (iv) False.

Question 5:

(a) This is FALSE, eg $x = 1$.

(b) This is TRUE as $(\forall x)o(x)$ is false, and any statement $F \to s$ is true.

(c) This is not defined as we have no predicate $n(y)$.

(d) This should say $d(x, y)$ not $q(x, y)$, in which case it is false, as for any x the value $y = x + 2$ does not satisfy.

Question 6:

See Coursework 4.

Section B

Question 7:

(a) See Coursework 4.

(b) Number of distinct pairs is ${}_nC_2$ which is $n(n-1)/2$. Largest possible sum is $n^2/5 - 1 + n^2/5 - 2 = 2n^2/5 - 3$.

(c) $p - m = n^2/2 - n/2 - 2n^2/5 + 3 = n^2/10 - n/2 + 3 = n(n-5)/10 + 3 > 0$ if $n > 5$.

(d) Suppose that every pair of numbers has a different sum. Then the number of possible sums is greater than the number of pairs, which contradicts part (c).

(e) If $a + b = a + c$ then $b = c$, which contradicts the two pairs being distinct.

(f) No, eg $n = 10$ take $\{1, 10\}$ or $\{2, 9\}$ or $\{3, 8\}$ or $\{4, 7\}, \ldots$.
(g) No because we have no idea which numbers have been chosen.

Question 8:

(a) i. $A \cup (B \backslash C)$.
ii. $((A \cup B) \cap C) \backslash (A \cap B)$.
iii. $A \backslash A$.

(b) (Omitted.)

(c) No, this is Russell's paradox.

(d) See Coursework 2.

Question 9:

(a)(i) A function f is injective if $f(x) = f(y)$ implies that $x = y$.

(ii) A function $f : X \longrightarrow Y$ is surjective if for all $y \in Y$ there exists $x \in X$ with $f(x) = y$.

(iii) A function is bijective if it is both injective and surjective.

(b) (i) f is injective, not surjective.

(ii) g is surjective, not injective (as $g(2) = 2 = g(4)$).

(iii) $h(x)$ is odd for all x so not surjective. $h(0) = 1 = h(-1)$ so not injective.

(c) Two sets have the same cardinality if there exists a bijection between them. A set X has cardinality $\aleph_0$ if there exists a bijection from $\mathbb{N}$ to X.

(d) See course notes.

(e) See course notes.
(f) $(p \wedge (\neg q)) \vee ((\neg p) \wedge q)$, verify with truth table as in Q3.