

Mathematics Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office (C123)** by **2pm on Thursday 22nd October**. Late submissions will be penalised. The final mark for this coursework will be out of 60, and include the marks obtained from the first two tutorial tests.

1. Simplify

$$\frac{x^4 + x^3 + 3x^2 - 5x + 7}{x^2(x^2 + 1)}$$

using partial fractions.

[10]

2. Find the domain and range of

$$f(x) = \sin\left(\frac{1}{x+1}\right) + \exp\left(\frac{1}{x+1}\right).$$

Does this function have an inverse? (Give reasons for your answer.)

[7]

3. (i) Show that

$$\cos(2\theta) = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}.$$

(ii) Find the general solution to $\cos 3\theta = \cos(5\theta - \frac{\pi}{7})$.

[8]

4. Find all solutions of

$$2 \cos 2\theta + 4 \sin \theta \cos \theta = \sqrt{2}$$

in the range $-\pi \leq \theta \leq \pi$.

[10]

5. Solve the equation

$$\log_3(x) = \log_9(|x + 6|).$$

[8]

6. Using the identity

$$(1 + x)^n(1 - x)^n = (1 - x^2)^n$$

deduce that if $2r \leq n$ then

$$\sum_{a=0}^{2r} (-1)^{r-a} \binom{n}{a} \binom{n}{2r-a} = \binom{n}{r}.$$

[7]