Mathematics Coursework 2

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (C123) by **2pm on Thursday 5th November**. Late submissions will be penalised. The final mark for this coursework will be out of 60, and include the marks obtained from the second pair of tutorial tests.

1. Solve the equation
$$\left|\frac{(x+1)}{(x-3)(x-5)}\right| < \frac{1}{2}.$$
[8]

2. (i) The points $(5,5)$ and $(-3,-1)$ are the ends of a diameter of the circle C with centre A. Write down the coordinates of A and determine an equation for C.
(ii) The line L with equation $y = 3x - 16$ meets C at the points P and Q. Find the coordinates for P and Q. Write down the gradients of the lines AP and AQ and hence calculate the area of the triangle APQ. **[10]**

3. (i) Differentiate the following equations with respect to x:
$$\text{(a)} \quad \frac{x^{\frac{1}{2}}\sin^2 x}{x^2+1} \qquad \text{(b)} \quad \ln(\cos(e^{x+1})).$$
(ii) If $y = (Ax+B)e^{-2x} + 2\sin 3x + \cos 3x$ where A and B are constants, show that
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y$$
is independent of A and B. **[8]**

4. (i) Calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the curve whose equation in *polar coordinates* is
$$r = 1 + \sin^2\theta.$$
(ii) Find the first and second derivatives (with respect to x) of the function
$$x = 3t^2 + 6t + 9 \qquad y = t^3 - t$$
[6]

5. Find and classify the stationary points of
$$y = e^x \sin(x)$$
with $-\pi < x < \frac{3\pi}{2}$. Also determine the points of inflexion and the global maxima and minima (if they exist) over this domain. **[10]**

6. Show that $y = \frac{1}{\sqrt{1+x^2}}$ satisfies

$$(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 0.$$

Use Leibnitz' theorem to show that for $n \geq 2$ we have

$$(1+x^2)\frac{\mathrm{d}^n y}{\mathrm{d}x^n} + (2n-1)x\frac{\mathrm{d}^{n-1} y}{\mathrm{d}x^{n-1}} + (n-1)^2\frac{\mathrm{d}^{n-2} y}{\mathrm{d}x^{n-2}} = 0.$$

[8]