Mathematics Coursework 3

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (C123) by **2pm on Thursday 19th November**. Late submissions will be penalised.

1. Calculate

$$\text{(a)} \quad \int \frac{x+2}{1-4x^2}\,dx \qquad \text{(b)} \quad \int_0^1 x^2 e^{-2x} dx \qquad \text{(c)} \quad \int \tan^2(x)\,dx.$$

[10]

2. (a) Show that $\sin(x) = \frac{2t}{1+t^2}$ where $t = \tan(\frac{x}{2})$.

   (b) If $t = \tan(\frac{x}{2})$ determine $\frac{\mathrm{d}x}{\mathrm{d}t}$ as a function of t.

   (c) Using (a) and (b) (or otherwise) calculate

$$\int \frac{1}{5\sin x + 3}\,dx.$$

[13]

3. Find the area between the curve

$$y = e^{-x}\sin 2x$$

   and the x-axis for $-\pi \leq x \leq \pi$. (Area below the x-axis should *not* be regarded as negative.) **[12]**

4. Find *from first principles* the Maclaurin series for $x\cos(3x)$ up to the term in x^4. **[8]**

5. Calculate

$$\text{(a)} \quad \lim_{x\to 0}\left(\frac{7x^3+8x^2+9x}{3x^2+\frac{5}{x}}\right) \qquad \text{(b)} \quad \lim_{x\to\infty}\left(\frac{4x^2+2x+1}{3+4x+5x^2}\right)$$

$$\text{(c)} \quad \lim_{x\to 0}\left(\frac{\sin^2 x\cos x}{x^2}\right).$$

[7]