Mathematics Answer Sheet 2

Sheet 3

1. $y = x^4$.

3. $\ln 2$, $\ln 3$.

4. (a) $p - \frac{1}{2}q$, (b) a^2b^{-3}.

5. (a) $\frac{1}{3}$, $\frac{2}{3}$, (b) 4, $16\sqrt{2}$.

6. $x < -6$, $x > -1$.

7. $2 < x < \frac{5}{2}$.

8. (a) $-1 < x < \frac{1}{2}$ or $x > 3$,
   (b) $u < -\ln 2$ or $u > \ln 3$.

9. $x = 3$, $y = -\frac{1}{3}$.

10. $x = -\frac{19}{2}$, $y = -\frac{23}{2}$, $z = \frac{15}{2}$.

11. No solution.

12. $x = \frac{3-5z}{4}$, $y = \frac{1-7z}{4}$ (or similar).

13. $2y = 3x - 29$, $(5, -7)$.

14. $12y + 5x = 22$, $k = -22$, 26.

15. $(-4, -5)$, $3\sqrt{5}$; $(1, 0)$, $\sqrt{5}$, $5\sqrt{2}$, (b) $3\sqrt{2}$.

16. $(\frac{24}{5}, \frac{32}{5})$.

Sheet 4

1. (a) $12x^2 - 4x - 2x^{-3}$.
   (b) $-3\sin(3x + 2)$.
   (c) $\frac{1-x^2}{(1+x^2)^2}$. (d) $1 + \ln x$.
   (e) $\frac{x}{\sqrt{1+x^2}}$.
   (f) $e^x \cos(x^2) - 2xe^x \sin(x^2)$.
   (g) $\frac{3}{(2x-1)^2} - \frac{2}{(x+2)^2}$.
   (h) $x^{e^x}(\frac{1}{x}e^x + \ln(x)e^x)$.
   (i) $-\sin(\tan(x^2))\sec^2(x^2)2x$.

2. (a) $3x^2 + 3y^2 + 6xy\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} - 4x^{-5}y^{-3} - 3x^{-4}y^{-4}\frac{dy}{dx} = 0$.
   (b) $-\sin(x)\sin(y) + \cos(x)\cos(y)\frac{dy}{dx} = 0$.
   (c) $-\sin(y\tan(x))(\frac{dy}{dx}\tan(x) + y\sec^2(x)) = (xy)^{-1}(y + x\frac{dy}{dx})$.
   (d) $\frac{(1+2\cos(y)\frac{dy}{dx})(x-y)+(x+2\sin(y)+4)(1-\frac{dy}{dx})}{(x-y)^2} = -\mathrm{cosec}^2(y)\frac{dy}{dx}$.
   (e) $\tan(x^{-1})\frac{dy}{dx} - y\sec^2(x^{-1})x^{-2} + \sec(y^{-1}) - x\sec(y^{-1})\tan(y^{-1})y^{-2}\frac{dy}{dx} = e^x$.

3. (a) $\frac{21t^2}{2t+2}$
   (b) $\mathrm{cosec}^3(t)$
   (c) $\frac{(3t^2+1)(1+2t^2)}{2t}$
   (d) $e^{-2t^2}\frac{\cos(t)-2t\sin(t)}{2t\cos(t)-\sin(t)}$
   (e) $-12t\sec^2(4t)\ln(t)^2$.

4. 8.

5. $f'(x) = -\frac{x}{2} + \frac{1}{x}$, $\beta = \sqrt{2}$.

7. (a) $-x^3\sin x + 15x^2\cos x + 60x\sin x - 60\cos x$,
   (b) $\frac{4}{x^3}$.