Mathematics Exercise Sheet 3

1. Obtain an expression for y in terms of x if

$$\ln(x^3) + \ln(xy) - \ln(y^2) = 0.$$

2. If x satisfies $3^x.4^{2x+1} = 6^{x+2}$, show that $x = \frac{\ln 9}{\ln 8}$.

3. Find x satisfying $e^{2x} - 5e^x + 6 = 0$.

4. If $\ln a = p$ and $\ln b = q$, express

   (a) $\ln\left(\frac{a}{\sqrt{b}}\right)$ in terms of p and q,

   (b) $\exp(2p - 3q)$ in terms of a and b.

5. Solve the equations

   (a) $\log_3(9x - 2) = 2 + 2\log_3 x$,

   (b) $\log_x 8 + \log_8 x = \frac{13}{6}$.

6. Find the set of values of x for which $2x(x + 3) > (x + 2)(x - 3)$.

7. Solve the inequality $\frac{x-3}{2-x} > 1$.

8. Solve the inequalities

   (a) $2x^3 - 5x^2 - 4x + 3 > 0$,

   (b) $2e^{3u} - 5e^{2u} - 4e^u + 3 > 0$.

9. Solve, if possible, the system of equations

$$2x + 3y = 5 \qquad \text{and} \qquad 3x - 6y = 11.$$

10. Solve, if possible, the system of equations

$$\begin{aligned} x &- 2y &- z &= 6 \\ 3x &- 6y &- 5z &= 3 \\ 2x &- y &+ z &= 0. \end{aligned}$$

11. Solve, if possible, the system of equations

$$\begin{aligned} x &+ 2y &+ 3z &= 1 \\ -2x &- y &+ 2z &= 2 \\ 3x &+ 3y &+ z &= 3. \end{aligned}$$

12. (∗) Find all possible solutions to the equations

$$\begin{aligned} x &+ y &+ 3z &= 1 \\ 3x &- y &+ 2z &= 2 \\ 8x &- 4y &+ 3z &= 5. \end{aligned}$$

13. Find the equation of the line through $(9, -1)$ perpendicular to the line $2x + 3y = -11$. Calculate the coordinates of the point where these two lines meet.

14. The straight line through $P(2, 1)$ and $Q(k, 11)$ has gradient $-\frac{5}{12}$. Find an equation for the line in terms of x and y, determine the value of k, and calculate the distance between P and Q.

15. Find the centres and radii of the circles

$$x^2 + y^2 + 8x + 10y - 4 = 0 \qquad \text{and} \qquad x^2 + y^2 - 2x - 4 = 0.$$

    Find also the distance between their centres and hence:

    (a) show that the circles intersect at right angles;

    (b) find the length of their common chord (i.e. the distance between their points of intersection).

16. The equation of a circle with centre C is

$$(x - 3)^2 + (y - 4)^2 = 9$$

    and O is the origin. The line joining O and C can be extended to meet the circle at P. Find the coordinates of P and show that the equation of the tangent to the circle at P is $3x + 4y = 40$.