

Mathematics Exercise Sheet 6

1. Calculate

$$(a) \int x \ln x \, dx \quad (b) \int x(\ln x)^2 \, dx.$$

2. Calculate

$$(a) \int \sqrt{3x+8} \, dx \quad (b) \int (3x+8)e^x \, dx \quad (c) \int 3x\sqrt{3x+8} \, dx.$$

3. (a) Given that $2y = x - \sin x \cos x$, show that $\frac{dy}{dx} = \sin^2 x$.

(b) Hence find

$$\int x \sin^2 x \, dx.$$

4. Calculate

$$\int_0^\pi x^2 \cos 3x \, dx.$$

5. Let I_n stand for the integral

$$\int x^n e^{2x} \, dx.$$

Use integration by parts to give a formula relating I_n to I_{n-1} . Use this result to determine I_4 .

6. Calculate

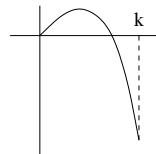
$$\int e^{3x} \sin 2x \, dx.$$

7. The curve with equation $y = e^{3x} + 1$ meets the line $y = 8$ at the point $(h, 8)$.

(a) Find h , giving your answer in terms of natural logarithms.

(b) Show that the area of the finite region enclosed by the curve with equation $y = e^{3x} + 1$, the x -axis, the y -axis, and the line $x = h$, is $2 + \frac{1}{3} \ln 7$.

8. The graph of $y = x(4 - x^2)$ is illustrated below for $x \geq 0$. Find the exact value of k for which the areas above and below the x -axis are equal.



9. Curves C and D have equations $y = \frac{1}{x}$ and $y = kx^2$ respectively, where k is a constant. The curves intersect at the point P , whose x -coordinate is $\frac{1}{2}$.

(a) Determine the value of k .

(b) Find the gradient of C at P .

(c) Calculate the area of the finite region bounded by C , D , the x -axis, and the line $x = 2$.

10. Find the volume generated when the area bounded by the curve $y = 4 - x^2$ and the line $y = 0$ is rotated about the y -axis.

11. Find the volume generated when the area defined by the inequalities $y \geq x^2 - 2x + 4$ and $y \leq 4$ is rotated about the line $y = 4$.