Ciphers and Number Theory 3

1. Calculate the greatest common divisor of each of the following pairs of numbers:

   (a) 35 and 82.

   (b) 51 and 238.

   (c) 154 and 84.

2. For each of the following equations, determine whether it is possible to find $u, v \in \mathbb{Z}$ so that they hold. If it is possible, solve the equations for u and v.

   (a) $35u + 82v = 1$.

   (b) $51u + 238v = 1$.

   (c) $51u + 238v = 17$.

   (d) $154u + 84v = 14$.

   (e) $154u + 84v = -42$.

3. Given $a, b \in \mathbb{N}$, suppose that $u, v \in \mathbb{Z}$ are a solution of the equation

   $$(a, b) = au + bv.$$

   Is the choice of u and v unique?

4. Show that if $a, b, c \in \mathbb{N}$ with $(a, b) = 1$ and c divides b then $(a, c) = 1$.

5. Find all solutions to the equation

   $$42x + 12y = 6$$

   with $x, y \in \mathbb{Z}$.