END OF CHAPTER EXERCISES

Chapter 18 : Pricing Interest Rate Derivatives

Financial Engineering : Derivatives And Risk Management

(Keith Cuthbertson, Dirk Nitzsche)

1. Why might you use Black's model to price a 1-year option on a T-bond where the underlying bond is a 15-year T-bond, but not where the underlying bond is a 1-year T-bond?

2. Price the following swaption using Black's model. The swaption is a 4-year swap, starting in 3 years, payments are annual and the principal in the swap is $100,000. The volatility of the swap rate is 20% p.a. It is a payer swaption with a strike (swap) rate of 7.5%. The yield curve is currently flat at 8% (continuous compounding).

3. Intuitively, how is the one-period interest rate lattice, in the no-arbitrage approach, consistent with the observed term structure of interest rates and their volatility? (Assume volatility is the same for all one-period rates in the lattice).

4. What advantages does a trinomial lattice have over a binomial lattice when used in pricing interest rate derivatives?

Data for Questions 5, 6 and 7
You are given the following data to be used in answering Q5 –Q7. The interest rate lattice over 3 periods is:

			16% (0.216)
		14% (0.36)	
	12% (0.6)		13% (0.432)
10%		10% (0.48)	
	8% (0.4)		9.0 (0.288)
		7% (0.16)	
			6% (0.064)

The risk neutral probability of an 'up' move is q = 0.6 and for a 'down' move is (1-q) = 0.4. The figures in parenthesis are the probability of reaching each node, times the number of ways to reach that node. They are therefore the BOPM terms :

$$q_k^n = \binom{n}{k} q^k (1-q)^{n-k}$$

Hence for n = 2, and k = 1 'up' moves, we are at the node 'ud' (or equivalently 'du')

and $$q_1^n = q_{ud}^n = \binom{2}{1}(0.6)^1(0.4)^1 = 0.48$$

similarly $$q_2^n = q_{uu}^n = \binom{2}{2}(0.6)^2(0.4)^0 = 0.36$$

5. What is the price of a two-year European cap with K_c = 10%?

6. How would your analysis in pricing the 2-year cap change, if the cap were an American style option ?

7. What is the 'price' (i.e. the FRA rate) for a 2-year FRA, where the actual cash payout occurs at t=3 ? This is sometimes called a 'delayed settlement FRA'.