END OF CHAPTER EXERCISES

Chapter 18 : Pricing Interest Rate Derivatives

Financial Engineering : Derivatives And Risk Management

(Keith Cuthbertson, Dirk Nitzsche)

1. Why might you use Black’s model to price a 1-year option on a T-bond where the underlying bond is a 15-year T-bond, but not where the underlying bond is a 1-year T-bond?

2. Price the following swaption using Black’s model. The swaption is a 4-year swap, starting in 3 years, payments are annual and the principal in the swap is $100,000. The volatility of the swap rate is 20% p.a. It is a payer swaption with a strike (swap) rate of 7.5%. The yield curve is currently flat at 8% (continuous compounding).

3. Intuitively, how is the one-period interest rate lattice, in the no-arbitrage approach, consistent with the observed term structure of interest rates and their volatility? (Assume volatility is the same for all one-period rates in the lattice).

4. What advantages does a trinomial lattice have over a binomial lattice when used in pricing interest rate derivatives?

Data for Questions 5, 6 and 7
You are given the following data to be used in answering Q5 –Q7. The interest rate lattice over 3 periods is:

16% (0.216)
14% (0.36) 13% (0.432)
12% (0.6) 10% (0.48) 10% (0.48)
8% (0.4) 8% (0.4) 9.0 (0.288)
7% (0.16) 7% (0.16) 6% (0.064)

The risk neutral probability of an ‘up’ move is q = 0.6 and for a ‘down’ move is (1-q) = 0.4. The figures in parenthesis are the probability of reaching each node, times the number of ways to reach that node. They are therefore the BOPM terms.
\[ q^n_k = \binom{n}{k} q^k (1 - q)^{n-k} \]

Hence for \( n = 2 \), and \( k = 1 \) ‘up’ moves, we are at the node ‘ud’ (or equivalently ‘du’)

and \( q^n_1 = q^n_{ud} = \binom{2}{1} (0.6)^1 (0.4)^1 = 0.48 \)

similarly \( q^n_2 = q^n_{uu} = \binom{2}{2} (0.6)^2 (0.4)^0 = 0.36 \)

5. What is the price of a two-year European cap with \( K_c = 10\% \)?

6. How would your analysis in pricing the 2-year cap change, if the cap were an American style option?

7. What is the ‘price’ (i.e. the FRA rate) for a 2-year FRA, where the actual cash payout occurs at \( t=3 \)? This is sometimes called a ‘delayed settlement FRA’.