

Chapter 3

Financial Functions in Excel

3.1 Introduction

This chapter develops the background for the financial functions in Excel that deal with loan repayments and other compound interest problems. For example if I borrow £5000 over three years at a fixed rate of interest how much do I pay back per month. Although the mathematical ideas behind such problems are quite straightforward some care needs to be employed when trying to implement the functions provided by Excel. Thus a thorough understanding of the mathematical formulas being used by Excel is essential in order to obtain and interpret correctly the answers given. Nearly all the work in compound interest is based on a single formula which in turn is derived from the sum of a geometric progression. We thus start with a few preliminary ideas.

3.2 Geometric Progression

A quantity is said to vary geometrically over a given set of time intervals if one value is some constant multiple of the previous value. Symbolically we have:

\textbf{Definition}

A geometric progression or series of \( n \) values is an ordered set of the form:

\[ a, \; ar, \; ar^2, \; ar^3 \ldots ar^{n-1} \]

where \( a \) is referred to as the first term and \( r \) the common ratio.

For example:

\[ 5, \; \frac{5}{2}, \; \frac{5}{4}, \; \frac{5}{8}, \ldots \; \frac{5}{2^{n-1}} \quad a = 5 \; \quad r = \frac{1}{2} \]

\[ 3, \; -\frac{3}{2}, \; \frac{3}{4}, \; -\frac{3}{8}, \ldots \; \frac{3}{2^{n-1}} \quad a = 3 \; \quad r = \frac{-1}{2} \]

We observe that in the second example the common ratio \( r \) is negative.

Usually we are interested in the sum of a geometric progression, a formula for which can be obtained as follows:
Let
\[ S_n = a + ar + ar^2 + ar^3 \ldots ar^{n-1} \]
then
\[ S_n - rS_n = (a + ar + ar^2 + \ldots ar^{n-1}) - (ar + ar^2 + ar^3 + \ldots ar^n) = a - ar^n \]
Thus we have
\[ S_n(1 - r) = a(1 - r^n) \quad \Rightarrow \quad S_n = a \left( \frac{1 - r^n}{1 - r} \right) \quad r \neq 1 \quad (3.1) \]
We note the following two points:

- If \( r = 1 \) the formula is not valid, however in this case \( S_n = a + a + \ldots + a \), \( n \) time and thus \( S_n = na \).
- In interest problems \(|r|\) is usually less than 1 and quite often \( n \) is large; we thus note the following result:

If \(|r| < 1\) then:
\[ \text{as} \quad n \rightarrow \infty \quad r^n \rightarrow 0 \quad \text{Thus Eq}(3.1) \quad \Rightarrow \quad S_n \rightarrow \frac{a}{1 - r} \]
Indeed \( \frac{a}{1 - r} \) is quite a common approximation for \( S_n \) when \( n \) is large and \( r \) is small.

### 3.3 Basic Compound Interest

The basic compound interest problem is:

If £\( A \) is invested for \( n \) periods at an interest rate of \( r \) per period, how much is the investment worth at the end of the \( n \) periods?

For example if I invest £1000 for 12 months compounded monthly at a rate of 1% at the end of each month how much do I have at the end of the year? In this example \( A = 1000, r = 0.01 \) and \( n = 12 \). Note that \( r \) is expressed as a fraction in the mathematics even though it is expressed as a percentage in the question. Fig 3.1 illustrates the growth of the initial investment \( A \) over the \( n \) periods as the interest is added at the end of each period. At the end of the first month you have your initial investment \( A \) plus the interest on the investment, \( rA \), giving a total of \( A(1 + r) \). This new amount is then used as the starting amount for the second period; thus at the end of the second interval we have \( A(1 + r) \) plus the interest on this amount, \( rA(1 + r) \). As we see this simplifies to give \( A(1 + r)^2 \). Continuing in this manner gives a total amount of \( A(1 + r)^n \) at the end of the \( n^{th} \) period.
In our simple example of £1000 being invested at 1% per month for a year the amount at the end of the 12 months = 1000(1 + 0.01)^{12} = £1126.83.

We note that the final amount is £26.83 more than if the compounding had been carried out only once at the end of the year at a rate of 12%. In all our problems it will be clear what the rate is and when we are applying it. We will not discuss the idea of effective interest rates or indeed the various ways in which banks express the interest rates in their advertising. However knowing mathematically what information is required to carry out the correct calculation will enable us to ask meaningful questions in any given situation.

Thus we summarise:

An amount $A$ invested at a rate $r$ for $n$ periods amounts to $A(1 + r)^n$.

### 3.4 Basic Investment Problem

If an amount $A$ is invested initially at a rate $r$ for $n$ periods and in addition we invest an amount $p$ at the end of each period what will the final amount be?

This simple problem represents someone who initially deposits an amount $A$ and then continues to invest, say at the end of each month, an amount $p$, with the interest being compounded at the rate $r$ at the end of each month.

As we will see it will be possible to adjust this basic problem to answer a variety of different questions.

With reference to fig 3.2 we see that at the end of the first period the initial investment has grown to $A(1 + r)$ \(^1\). Adding to this the regular investment $p$ gives a total of

\[ S_1 = A(1 + r) + p \]

at the start of the second period.

---

\(^1\)Note that it is true in general that after compounding over one period the amount at the end of the period is the amount at the start of the period multiplied by $(1 + r)$
CHAPTER 3. FINANCIAL FUNCTIONS IN EXCEL

Figure 3.2: Shows the detail of compounding after periods 1 and 2 with an initial investment \(A\) and subsequent investments \(p\) at the end of each period.

At the end of the second period this amount will have grown to its original value multiplied by \((1 + r)\). Adding in our next regular payment \(p\) means that we start the third period with:

\[
S_2 = \{A(1 + r) + p\}(1 + r) + p = A(1 + r)^2 + p(1 + r) + p
\]

Carrying out this process once more gives:

\[
S_3 = \left\{A(1 + r)^2 + p(1 + r) + p\right\}(1 + r) + p = A(1 + r)^3 + p(1 + r)^2 + p(1 + r) + p
\]

From this we can deduce the \(n^{th}\) case:

\[
S_n = A(1 + r)^n + p(1 + r)^{n-2} + p(1 + r)^{n-2} + \ldots + p
\]

After the first term we note that we have the sum of a geometric progression which with reference to Eq(3.1), \(a = p\) and the common ratio = \((1 + r)\) gives:

\[
S_n = A(1 + r)^n + p\left\{\frac{1-(1 + r)^{n-1}}{1-(1 + r)}\right\} = A(1 + r)^n + p\left(\frac{(1 + r)^n - 1}{r}\right)
\]

Thus we summarise:

<table>
<thead>
<tr>
<th>period</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>(A)</td>
<td>(A(1 + r) + p)</td>
<td>({A(1 + r) + p}(1 + r) + p)</td>
<td>(S_n)</td>
</tr>
<tr>
<td>(S_0)</td>
<td>(S_1)</td>
<td>(S_2 = A(1 + r)^2 + p(1 + r) + p)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An initial investment \(A\) and regular investment \(p\) at the end of each period, compounded at the rate \(r\) per period, becomes after \(n\) periods:

\[
A(1 + r)^n + p\left(\frac{(1 + r)^n - 1}{r}\right)
\]

Example 3.1

I initially invest £1000 in a saving scheme and then at the end of each month I invest an extra £50. If the interest rate is 0.5% per month and I continue this process for two years, how much will my saving be worth.
Substituting directly into Eq(3.2) for $A = 1000$, $p = 50$, $r = 0.005$ and $n = 24$ we obtain the final amount:

$$\text{final amount} = 1000(1 + 0.005)^{24} + 50 \left\{ \frac{(1 + 0.005)^{24} - 1}{0.005} \right\} = £2398.76$$

### 3.5 Basic Financial Worksheet Functions in Excel

In Example 3.1 it is clear how we should enter the parameters $A$, $p$, $r$, and $n$, however to use the worksheet functions in Excel we have to adopt a sign convention.

**Excel’s sign convention**

Money that flows away from you is given a negative sign and money that flows towards you is given a positive sign. A more concrete way of looking at this is, if you have to put your hand in your pocket and pay money out then it carries a minus sign, whereas on the other hand if money is put into your pocket then it carries a positive sign. In Example 3.1 we make an initial investment, thus in the Excel functions we would enter $-1000$ and not $+1000$. Similarly we make a monthly payment thus we would enter $-50$ into the Excel functions and not $+50$. The final amount is paid back to you thus Excel will yield a positive answer.

Thus denoting the final or future value by $FV$, monthly payments by $PMT$ and the initial or present value of our investment by $PV$ the sign convention would require:

- $A = -PV$ since the initial amount is paid out to the Bank
- $p = -PMT$ since the payments are paid out to the Bank
- $S_n = FV$ since the future value is paid back to you.

With this in mind and substituting into Eq(3.2) Excel is in fact using the formula:

$$FV = -PV(1 + r)^n - PMT \left\{ \frac{(1 + r)^n - 1}{r} \right\}$$

### Excel’s Future Value Function

$$= FV(r, n, PMT, PV, \text{type})$$

The parameters are as in Eq(3.3) with the additional parameter ”type”. If type is set equal to zero then the formula is as in Eq(3.3) with the payments being made at the end of each period. If type is set equal to 1 then it is assumed that the payments are made at the start of each period and thus the formula is amended accordingly.

To solve Example 3.1 using Excel we would enter into the worksheet:

$$= FV(0.005, 24, -50, -1000, 0)$$
or alternatively the rate of 0.005 could be entered as 0.5%. This gives £2398.76 as before.

**Excel’s Present Value Function**

\[ PV(r, n, PMT, FV, \text{type}) \]

This is simply Eq(3.3) rearranged to make PV the subject of the formula giving:

\[
PV = -\left(\frac{FV + PMT \left\{\frac{(1+r)^n - 1}{r}\right\}}{(1+r)^n - 1}\right)
\]

**Example 3.2**

If I wish to accumulate £5000 in four years time by depositing £75 per month in a fixed rate account with interest rate of 0.4% per month, what initial investment must I also make.

Here the accumulated amount of £5000 is the FV which is paid to us, thus it is entered as a positive quantity. We are making payments of £75 thus we enter PMT as -75. Four years is 48 periods thus \(n = 48\) and the interest rate is entered as 0.004 or 0.4%. Thus we enter:

\[ = PV(0.004, 48, -75, 5000, 0) \]

This gives -858.55, the minus sign indicating that we must pay out, that is to say deposit £858.55, at the start of the investment in order to reach £5000 by the end.

**Excel’s Payment Function**

\[ PMT(r, n, PV, FV, \text{type}) \]

Again this is a simply rearrangement of Eq(3.3), making PMT the subject of the formula gives:

\[
PMT = -\left\{FV + PV \left(\frac{1}{1+r}\right)^n\right\} \times \left\{\frac{r}{(1+r)^n - 1}\right\}
\]

The sort of question that you may wish to answer is typically the following concerning the repayment of a loan.

**Example 3.3**

How much will the monthly repayments be if I borrow £100,000 over 20 years with an effective monthly interest rate is 0.5%.

In this problem the final value, FV, is zero, since we must pay back all the loan and interest by the end of the period. The loan is paid to us at the start, so the present value, PV is +100,000. The number of payments is 240 and we assume that we pay at the end of each month thus type=0.

Thus we need to enter:

\[ = PMT(0.5\%, 240, 100000, 0, 0) \]
This gives $-716.43$. As expected the result is negative since we are paying out money each month.

**Excel’s Number of Periods Function**

$$\text{=NPER}(r, \text{PMT}, \text{PV}, \text{FV}, \text{type})$$

This function calculates the number of periods in a financial problem, in the case of repaying a loan this is the number of repayments, as in Example 3.3 above. Again a simple rearrangement of Eq(3.3) making \(n\) the subject of the formula gives:

$$n = \frac{1}{\ln(1 + r)} \ln \left( \frac{\text{PMT}}{\text{FV}} - \frac{\text{PMT}}{\text{PMT} + \text{PV}} \right)$$

A simple example to illustrate this would be:

**Example 3.4**

How long would it take me to pay off a loan of £10,000 at a rate of 0.5% per month if I can afford to pay £100 per month. The loan is paid to us at the start so \(\text{PV}=+10000\), at the end of the loan the final value is zero so we set \(\text{FV}=0\), since we pay out the payments \(\text{PMT}=−100\) and as usual assuming that we are making the payments at the end of the month \(\text{type}=0\). Thus we enter:

$$\text{=NPER}(0.5\%, −100, 10000, 0, 0)$$

This gives 138.98, which means we need to make 138 payments of £100 plus a final payment of less than normal or alternatively we make 138 payments with the 138th payment being larger than normal. We can investigate the two options by using the FV function:

- If we make 139 payments of £100 then we will overpay, that is to say the future value will be positive instead of zero. We need to find the future value of the loan after 139 payment of £100 and then adjust the final payment accordingly. Using the FV function:

  $$\text{=FV}(0.5\%, 139, −100, 10000, 0)$$

  gives 2.42, that is to say we will overpay by £2.42. Thus the final payment (the 139th) should be £97.58

- If we only make 138 payments of £100 then how much of the loan will still be outstanding? Changing the number of payments in the FV function gives:

  $$\text{=FV}(0.5\%, 138, −100, 10000, 0)$$

  which gives −97.09. Thus we have an under payment of £97.09 which means our final (the 138th) payment must be £100+£97.09=£197.09.
**Excel’s rate Function**

\[ = \text{RATE}(n, \text{PMT}, \text{PV}, \text{FV}, \text{type}, \text{guess}) \]

This function calculates the value of \( r \) from Eq(3.3), however it is not possible to make \( r \) the subject of the formula and hence we are unable to obtain an explicit formula for \( r \) in terms of the other parameters. Excel solves Eq(3.3) using an iterative scheme with a starting value equal to ”guess”, which is the final parameter of the RATE function. Although not certain, it is most likely that Excel uses Newton’s method or one of its variations to implement the iterative scheme. In detail Excel uses an iterative scheme with first value ”guess” to solve:

\[
f(r) = \text{FV} + \text{PV}(1 + r)^n + \text{PMT} \left\{ \frac{(1 + r)^n - 1}{r} \right\} = 0
\]

In most cases setting ”guess” equal to zero works well, indeed omitting the ”guess” parameter completely will still work and Excel will assume by default that it is zero.

**Example 3.5**

I borrow £1000 over 1 year making payments of £100 per month at the end of each month. What is the monthly interest rate? Since we borrow £1000 the present value \( PV=1000 \); the payments have a negative sign since they are paid out thus \( PMT=-100 \); the number of payments \( =12 \); type is set equal to zero since the payments are at the end of each month; guess=0 as recommended. Thus we enter:

\[ = \text{RATE}(12, -100, 1000, 0, 0, 0) \]

which gives 2.92%.

### 3.6 Further Financial Worksheet Functions in Excel

Although Excel does have many worksheet functions we now consider just four more that are to do with the amount of interest paid in any period of a given loan. One may wish to know this for tax purpose.

**Interest and Principal**

When paying back a loan each payment can be thought of as consisting of two parts:

- Payment of Interest
- Payment towards paying off the original loan

Excel provides us with functions to calculate these values for a single payment at any point in the repayment process. It also provides a function to calculate the total amount of interest paid over a given period of the loan. Similarly there is a function that calculates the total amount of principal paid back over a given period.
To see precisely what these values are we construct a repayment diagram similar to the investment diagram in Fig 3.2 (replace \( p \) with \(-p\)) and add to the diagram the interest due at the end of each month. From Fig 3.3 we see that after the first period, at \( i = 1 \),

\[
\begin{array}{cccccc}
\text{period} & 0 & 1 & 2 & 3 & \ldots & n \\
\hline
\text{Amount owed} & A & A(1+r) - p & A(1+r)^2 - p(1+r) - p & \cdots \\
\text{Interest due} & 0 & Ar & \{A(1+r) - p\}r & \{A(1+r)^2 - p(1+r) - p\}r \\
\end{array}
\]

Figure 3.3: Shows the amount outstanding after each payment and the amount of interest due at each stage.

the amount of interest is \( Ar \) and the amount owing is the original loan \( A \) plus the interest \( Ar \) less the payment \( p \). It is this amount, \( A(1+r) - p \), on which interest is charged over the next period. Thus at \( i = 2 \) the interest due is this amount times \( r \), namely \( \{A(1+r) - p\}r \). The diagram shows this process for one more step. In general we deduce that at the end of the \( i^{th} \) period the interest that is due is given by:

\[
\text{Interest} = \{A(1+r)^{i-1} - p(1+r)^{i-2} - p(1+r)^{i-2} - \ldots - p\}r
\]

After the first term which involves \( A \) we have a geometric progression with \((i-1)\) terms, first term \(-p\) and common ratio \((1+r)\), thus summing this progression gives:

\[
\text{Interest} = Ar(1+r)^{i-1} - pr\left\{\frac{1 - (1+r)^{i-1}}{1 - (1+r)}\right\} = (1+r)^{i-1}\{Ar - p\} + p \quad (3.4)
\]

Since this represents the amount of interest paid off within the single payment \( p \) after the \( i^{th} \) period the amount of principal paid off is simply \( p \) minus this value. Hence

\[
\text{Principal paid} = p - \text{Interest} = (1+r)^{i-1}\{p - Ar\}
\]

**Excel’s Interest and Principal Payment Functions**

The function that calculates the amount of interest in any given payment is:

\[
= \text{IPMT}(r, i, n, PV, FV, type)
\]

and the function that calculates the amount of principal paid off in any given payment is:

\[
= \text{PPMT}(r, i, n, PV, FV, type)
\]

To convert the mathematical formula in Eq(3.4) to the Excel formula we need to make reference to Excel’s sign convention. Since in Eq(3.4) the amount \( A \) is a loan it will be
input in Excel as \(+PV\), however since the repayments are moving away from us the \(p\) will be replaced with \(-PMT\), interest with \(-IPMT\) and the principal with \(-PPMT\).

Thus in Excel:

\[
\text{IPMT} = - \left\{ (1 + r)^{i-1}\{PVr + PMT\} - PMT \right\} \quad \text{and} \quad \text{PPMT} = PMT - IPMT
\]

We note that the two functions IPMT and PPMT contain \(n\) and \(FV\) these are used internally by Excel to calculate \(PMT\) using the function =PMT\((r,n,PV,FV,\text{type})\). The type parameter is as usual set equal to zero for payments at the ends of the periods and one for payments at the start. (formula not covered here)

In the case of paying off the whole of a loan we set \(FV=0\), however the functions do allow us the flexibility to specify a different final value.

**Example 3.6**

If I borrow £20000 making monthly repayments at the end of each month at a monthly rate of 2/3\% over 3 years then

(a) How much interest and principal do I pay in the final payment?

(b) How much interest and principal do I pay in the first payment?

(a) For the interest in the final payment we set \(i = 36\) and enter

\[
=\text{IPMT}\((2/3)\% , 36, 36, 20000, 0, 0)\]

which gives £4.15. The minus sign indicating, as expected, that we pay £4.15 of interest in our final payment.

For the principal in the final payment we again set \(i = 36\) and enter

\[
=\text{PPMT}\((2/3)\% , 36, 36, 20000, 0, 0)\]

which gives £622.58, again the minus sign indicating that we are paying out this amount, in this case toward the principal.

(b) To calculate the amount of interest and principal in the first payment we enter the two functions with \(i\) set equal to 1.

Thus:

\[
=\text{IPMT}\((2/3)\% , 1, 36, 20000, 0, 0)\]

gives £133.33 indicating that we pay £133.33 in interest in our first payment and

\[
=\text{PPMT}\((2/3)\% , 1, 36, 20000, 0, 0)\]

gives £493.39 indicating that we pay £493.39 off the loan in our first payment.
Excel’s Cumulative Interest and Principal Payment Functions

Excel has two functions that deal with the amount of interest paid and the amount of principal paid off in a sequence of consecutive payments within the loan period. To do this Excel introduces two new parameter to replace the \( i \) parameter above. The parameter ”start” indicates the period in which we start adding the amounts and the parameter ”end” indicates the period in which we stop. These values are inclusive, thus to calculate the total interest paid in the first twelve payments of a loan, ”start” would be set equal to 1 and ”end” equal to 12. If the payments are monthly then to calculate the total interest paid in the second year ”start” would be set equal to 13 and ”end” equal to 24. The functions that do these calculations are:

\[
\text{=CUMIPMT}\left(r, n, PV, \text{"start"}, \text{"end"}, \text{type}\right) \quad \text{cumulative interest function}
\]

and

\[
\text{=CUMPRINC}\left(r, n, PV, \text{"start"}, \text{"end"}, \text{type}\right) \quad \text{cumulative principal function}
\]

Since there is no FV parameter in these functions their use is restricted to problems where the loan in completely paid off at the end of the \( n \) payments. That is to say FV has been internally set equal to zero and we have no way of changing it.

Finally to use these function we must ”add-in” the Analysis ToolPak before we enter them into the spreadsheet.

To enable the analysis add-in:

- Select Tools from the top menu
- Select Add-Ins
- Tick Analysis ToolPak

Example 3.7

With reference to Example 3.6
(a) calculate the total amount of interest paid during the final year using CUMIPMT.
(b) Check that the total amount of principal is indeed paid back using the CUMPRINC function.

(a) Enter:

\[
\text{=CUMIPMT}\left(\left(\frac{2}{3}\right)\%, 36, 20000, 25, 36, 0\right)
\]

Note that the last year is from month 25 to 36 inclusive. This gives −316.01 indicating that £316.01 of interest is paid during the final year.

(b) Use CUMPRINC to check that the scheme does indeed pay of the whole loan:
Entering

\[ =\text{CUMPRINC}\left((2/3)\%,\ 36,\ 20000,\ 1,\ 36,\ 0\right) \]

with "start" equal to 1 and "end" equal to 36 gives \(-20000\) indicating that the complete loan of £20000 has been paid.