MA3606 DIFFERENTIAL EQUATIONS - EXERCISES 1

First order equations

1. Solve the initial value problems:

   (i) $y' + y = e^x, \quad y(0) = 0,$

   (ii) $y' + 2xy = x, \quad y(0) = 0.$

2. Use the method of separation of the variables to find a solution of the initial value problem

$$y' = x\sqrt{1-y^2}, \quad y(0) = 1.$$

   Show that another solution is given by $y(x) = 1$. Does this contradict the uniqueness theorem?

3. Consider the initial value problem $\quad y' = 1 + xy^2, \quad y(0) = 0.$

   (a) Write the differential equation as an integral equation $\quad y(x) = L[x, y(x)]$.

   (b) Define the Picard iterates by $y_{n+1}(x) = L[x, y_n(x)]$. Taking $y_0(x) = 0$, find $y_1(x)$, $y_2(x)$ and $y_3(x)$.

Second order equations: homogeneous

4. Determine the largest interval on which the existence and uniqueness theorem ensures a solution to each of the following initial value problems:

   (i) $(1+x^2)y'' + xy' - y = \tan x, \quad y(0) = y_0, \ y'(0) = y_0',$

   (ii) $e^x y'' + \dfrac{1}{(x^2-3)} y' + y = \ln x, \quad y(1) = y_0, \ y'(1) = y_0'.$

5. Find a fundamental set of solutions for each of the following equations by using the trial function $y = e^{mx}$ to find solutions and showing that the Wronskian of these solutions is not zero:

   (i) $y'' + 5y' + 4y = 0,$ (ii) $y'' + 4y' + 5y = 0,$ (iii) $y'' - 4y = 0.$

6. Verify, by substitution, that $y_1(x) = x$ is a solution of the differential equation

$$(1-x^2)y'' - 2xy' + 2y = 0, \qquad -1 < x < 1,$$

   and find a second linearly independent solution.