

MA3606 DIFFERENTIAL EQUATIONS - EXERCISES 3

Second order equations: self-adjoint form and influence function

1. Write each of the following differential equations in self-adjoint form:

(i) $x^2y'' + xy' + (x^2 - p^2)y = 0$, p constant,

(ii) $y'' - 2xy' + 2y = 0$,

(iii) $(x + 1)y'' - y' + 2xy = 0$.

2. Find the influence function for the initial value problem

$$y'' - y = f(x), \quad x > 0,$$

$$y(0) = y'(0) = 0.$$

Hence express the solution as an integral.

3. Solve the initial value problem

$$y'' + y' - 2y = e^x \quad x > 0,$$

$$y(0) = 1, \quad y'(0) = 0,$$

by using the influence function.

Eigenvalues and eigenfunctions

4. Find the eigenvalues and corresponding eigenfunctions of the homogeneous boundary value problem

$$y'' + \lambda y = 0, \quad 0 < x < L,$$

$$y'(0) = y'(L) = 0.$$

5. Find the eigenvalues and corresponding eigenfunctions of the homogeneous boundary value problem

$$y'' + \lambda y = 0, \quad 0 < x < 2\pi,$$

$$y(0) = y(2\pi), \quad y'(0) = y'(2\pi).$$

6. Find the eigenvalues and corresponding eigenfunctions of the homogeneous boundary value problem

$$y'' + 4y' + (4 + \lambda)y = 0, \quad 0 < x < a,$$

$$y(0) = y(a) = 0.$$