MA3606 DIFFERENTIAL EQUATIONS - EXERCISES 5

Linear systems: constant coefficent equations

1. Find the general solution of each of the following systems of equations by evaluating the eigenvalues and eigenvectors of the system matrix:

(i) $y_1' = 2y_1 + 3y_2$
$y_2' = 2y_1 + y_2$

(ii) $y_1' = -2y_1 + 2y_2$
$y_2' = -y_1$

2. For each of the following systems, find a coordinate transformation $\underline{y} = P\underline{z}$ which transforms the system to canonical form and hence find the particular solution which satisfies the initial condition $\underline{x}(0) = (1,-1)$:

(i) $y_1' = y_2$
$y_2' = -2y_1 + 3y_2$

(ii) $y_1' = 9y_1 + 4y_2$
$y_2' = -9y_1 - 3y_2$

3. Find the general solution of each of the following systems by transforming the system matrix to canonical form:

(i) $y_1' = 2y_1 + 3y_2 + t$
$y_2' = 2y_1 + y_2 + 4t$

(ii) $y_1' = 4y_1 + 2y_2 - 7e^{-2x}$
$y_2' = 3y_1 - y_2$

Linear systems: general equations

4. Consider the system

$$y_1' = 2y_1 + 3y_2$$
$$y_2' = 2y_1 + y_2$$

(a) Find a fundamental matrix for the system. (*Use solutions obtained in Question 1.*)
(b) Hence evaluate a transition matrix for the system.
(c) Hence find the solution such that $\underline{y}(0) = (-1,2)$.

5. Verify that a transition matrix for the system

$$\underline{y'} = \begin{pmatrix} 0 & x \\ -x & 0 \end{pmatrix} \underline{y}$$

is given by $$M(x,x_0) = \begin{pmatrix} \cos\frac{1}{2}(x^2 - x_0^2) & \sin\frac{1}{2}(x^2 - x_0^2) \\ -\sin\frac{1}{2}(x^2 - x_0^2) & \cos\frac{1}{2}(x^2 - x_0^2) \end{pmatrix}.$$

Hence find the solution of the system which satisfies $\underline{y}(0) = (2,-1)$.

6. Prove the transition property for a transition matrix.
Verify the property for the transition matrix defined in Question 5.