DIFFERENTIAL EQUATIONS (302) - EXERCISES 6

First order partial differential equations

1. Find the general solution of the equation $u_x + 2xy^2 u_y = 0, \quad x, y > 0.$

2. Find the solution of the equation $xu_x + 2xe^y u_y + u = 0, \quad x, y > 0,$

   subject to the initial condition $u(x,0) = x.$

3. Find the solution of the partial differential equation

$$(x+1)u_x + (2x+3)y\,u_y = 0, \quad x, y > 0,$$

   which satisfies the condition $u(0,y) = y^2.$

Second order partial differential equations: classification and canonical form

4. Classify each of the following equations as hyperbolic, parabolic or elliptic:

   (i) $u_{xx} - u_{xy} - 2u_{yy} = 0, \quad x, y \in \mathbf{R}^2,$

   (ii) $2u_{xx} + 4u_{xy} + 3u_{yy} - 5u = 0, \quad x, y \in \mathbf{R}^2,$

   (iii) $yu_{xx} - 2u_{xy} + e^x u_{yy} - 5u = 0, \quad x, y \in \mathbf{R}^2.$

5. Classify the equation

$$u_{xx} + 6u_{xy} + 9u_{yy} + 3u_x + 9u_y = 0$$

   as hyperbolic, parabolic or elliptic.

   Show that a set of characteristic coordinates is given by $\xi = 3x - y, \ \eta = x.$

   Using these coordinates, transform the equation to canonical form and thus find its general solution.

6. Show that the equation

$$u_{xx} - (x-1)u_{xy} - xu_{yy} - u_x + xu_y = 0, \quad x > 0, \ -\infty < y < \infty,$$

   is hyperbolic and find the equations of the characteristics.

   Define a set of characteristic coordinates and transform the equation to canonical form.