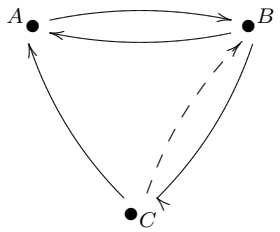
DISCRETE MATHEMATICS, EXERCISES SHEET 2

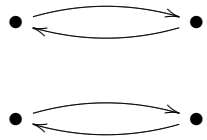
(1) Consider the following directed graph (seen in Exercises Sheet 1, Problem 2(b) and 4).



   (a) Write down two different adjacency matrices of the digraph. Show that they are similar matrices.
   (b) Choose an adjacency matrix W from part (a) and work out the associated stochastic matrix $\overline{W}$. Find an invariant measure for $\overline{W}$. (You can do this from scratch, or use the result of Exercises Sheet 1, Problem 4.)
   (c) Consider a modification of random surfing in which surfers are twice as likely to follow 'dotted' links as to follow 'normal' links. Write down a stochastic matrix for this modified Markov chain. In the modified version, what is the probability of surfing from page B to page C in three steps?

(2) Give an example of a model of the web with 5 pages A, B, C, D, E with the following properties:
   (a) Each page has at least one outlink.
   (b) In the long run page E has no traffic.
   (c) The pages are ranked in the following order: $A = B > C = D > E$.

(3) Consider the following 4-vertex digraph:



   Write down the adjacency matrix W and the associated row stochastic matrix $\overline{W}$. Show that an invariant measure exists for $\overline{W}$ but that it is not unique. What are the implications for the ranking of pages in the associated model of the web?

(4) Let M be a square matrix and let λ be an eigenvalue of M.
   (a) Show that λ^k is an eigenvalue of M^k;
   (b) Show that, for t any complex number, $t\lambda$ is an eigenvalue of tM.