

DISCRETE MATHEMATICS, PROBLEMS SHEET 3

- (1) We have seen that any stochastic matrix M has an invariant measure. Show that if M has at least two different invariant measures then it has infinitely many.
- (2) Let M be a 4×4 matrix with eigenvalues 2 and -1 . List all the possibilities for the characteristic polynomial of M . List all the possibilities for Jordan form of M .
- (3) You are given m matrices, each with characteristic polynomial $(x - 2)^2(x - 1)^2(x - 4)^5$, such that no two are similar. What is the largest m can be?
- (4) Let

$$M = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}.$$

Find an explicit formula for the entries of the matrix M^k .

- (5) Show that every 2×2 matrix of the form

$$\begin{pmatrix} \lambda & y \\ 0 & \lambda \end{pmatrix},$$

with $y \neq 0$, is conjugate to a matrix of the same form, but with $y = 1$.

- (6)
 - (a) Let $M = J_4(\lambda)$, the Jordan block of size 4. Work out M^2, M^3, M^4 .
 - (b) Give an explicit description of $J_n(\lambda)^k$, an arbitrary power of an arbitrary Jordan block.
 - (c) Show that the sequence of powers $J_n(\lambda), J_n(\lambda)^2, \dots$ converges to the zero matrix when $|\lambda| < 1$.