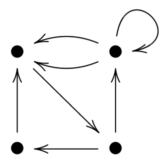
DISCRETE MATHEMATICS, SOLUTIONS SHEET 1

(1) The matrix is the adjacency matrix of the digraph



with respect to ordering the vertices clockwise starting from the upper left.

(2) The vertices of the digraphs represent web pages, and the edges between them links. The first digraph is symmetric with respect to a cyclic permutation of the vertices. Hence the 3 pages should be equally important. To discuss the second digraph, let us label its vertices (or the webpages they represent) A, B, and C, starting at the upper left and working clockwise. All surfers visiting A will be 'forced' to progress to B at the next interval. On the other hand B has visitors coming from C as well. So B is more important than A. Next, A and C receive equal traffic directly from B, but C also sends some surfers along to A – hence A is more important than C.

(3) For the second digraph, there are six possibilities for the adjacency matrix:

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

corresponding to the $6 = 3!$ ways of ordering the three vertices. A nice way to see this is to consider adding one more edge to the digraph, so that all vertices have edges to every other vertex. The resulting complete digraph has the adjacency matrix,

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

independently of how the vertices are ordered, and the 6 adjacency matrices for the original digraph are obtained by changing a single entry from 1 to 0.

For the first digraph there are only 2 possibilities,

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

because of the symmetry of the digraph mentioned above.

(4) Label the three pages A, B and C, starting from the upper left and going clockwise. After one 'click', the 12 surfers at A all go to B; half of the 12 at B go to A and the other half to C; and those at C split equally as well, 6 moving to A and 6 to B. The final tally: $A : 12, B : 18, C : 6$.

Now let us suppose that there are x,y and z surfers at pages A, B and C, respectively. By the same reasoning as above, after one iteration of random surfer there will be $\frac{1}{2}y+\frac{1}{2}z$,

$x + \frac{1}{2}z$ and $\frac{1}{2}y$ surfers at pages A, B and C. So if this represents a stable configuration of surfers, we deduce that $x = \frac{1}{2}y + \frac{1}{2}z$, $y = x + \frac{1}{2}z$ and $z = \frac{1}{2}y$. From the last equation we have $y = 2z$, and then plugging in to the first, we obtain $x = \frac{3}{2}z$. Since we also assume that there are 36 surfers in total, i.e., $x + y + z = 36$, we get $\frac{9}{2}z = 36$. So $z = 8$, $y = 16$ and $x = 12$. Hence in the long run we have 12, 16 and 8 surfers at pages A, B and C.