# MA3615 Groups and Symmetry: Coursework 1

This is an assessed coursework. Solutions should be handed in to the Mathematics general office (C123) by **3pm on Wednesday 3rd March 2010**. Late submission will be penalized.

1. Which of the following are groups? Justify your answers.

   (a) $A = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in \mathbb{R}\}$ with multiplication of matrices.

   (b) $B = 3\mathbb{Z}$ the set of all integers which are divisible by 3 with addition.

   (c) $C = \{e, (1,2,3,4), (1,3)(2,4), (1,4,3,2)\}$ with composition of permutations.

   For those which are groups, decide whether they are abelian or not. Justify your answers. [15]

2. (a) Describe the elements of the group $G$ of all rotational symmetries of a 4-pyramid (its base is a square).

   (b) Show that the group $G$ is isomorphic to the group $H$ given by the set $H = \{1, 2, 3, 4\}$ with multiplication modulo 5.

   (c) By labelling some of the vertices of the 4-pyramid, find an isomorphism with the group $C$ given in question 1.

   (d) Can the group $G$ be isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$? Justify your answer.

   [16]

3. Let $(G, *)$ be a finite group with identity $e$.

   (a) Suppose that $|G| = 6$ and let $g \in G$ be an element with $g^{13} = e$. What can you deduce about $g$?

   (b) Suppose that $|G| = 17$. List all the subgroups of $G$.

   [8]

4. Let $G = \{a, b, c, d\}$. We define three different 'multiplications' on $G$, namely $*$, $\dagger$ and $\times$ by giving the following tables:

| $*$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $b$ | $a$ | $d$ | $c$ |
| $b$ | $a$ | $b$ | $c$ | $d$ |
| $c$ | $d$ | $c$ | $b$ | $a$ |
| $d$ | $c$ | $d$ | $a$ | $b$ |

| $\dagger$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $a$ | $b$ | $c$ | $d$ |
| $b$ | $b$ | $a$ | $d$ | $c$ |
| $c$ | $c$ | $d$ | $a$ | $b$ |
| $d$ | $d$ | $a$ | $c$ | $a$ |

| $\times$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $b$ | $d$ | $a$ | $c$ |
| $b$ | $d$ | $c$ | $b$ | $a$ |
| $c$ | $a$ | $b$ | $c$ | $d$ |
| $d$ | $c$ | $a$ | $d$ | $b$ |

   Which of these multiplications turn $G$ into a group? For those which don't, explain why not. For each of those which do, find a group seen at the lecture or on the exercise sheets which is isomorphic to it. [15]

5. Let $K = \{g, h, i, j, k, l\}$ be a group (with multiplication $*$) and let

$$\phi : S_3 \longrightarrow K$$

be an isomorphism with $\phi(1,2) = g$, $\phi(2,3) = h$. Assume further that $g^2 = i$, $g * h = j$ and $k$ has order 3. Find the cayley table for $K$. [15]

6. List all the subgroups of $\mathbb{Z}_{12}$. [15]

7. Let $D_8 = \{e, r, r^2, r^3, s, rs, r^2s, r^3s\}$ be the group of all symmetries of a square. Find the left and right cosets of the subgroup $H$ in $D_8$ where

   (a) $H = \langle r^3 \rangle$

   (b) $H = \langle r^2 s \rangle$

   (c) $H = \langle r^2, s \rangle$

   (d) $H = \langle r^3, s \rangle$.

[16]