MA3615 Groups and Symmetry: Coursework

This is an assessed coursework. Solutions should be handed in to the General office (C108) by **4pm on 24 March 2011**. Late submission will be penalized.

1. Decide whether the following statements are true or false. Justify your answers.

   (a) There exists a subgroup H of A_5 with $|H| = 9$.

   (b) Every element $g \in S_4$ satisfies $g^{48} = e$.

   (c) The group $\mathbb{Z}_2 \times \mathbb{Z}_2$ is generated by one element.

   (d) The group $\mathbb{Z}_2 \times \mathbb{Z}_3$ is generated by one element.

   (e) The group D_6 has a proper non-trivial normal subgroup.

   (f) The group D_6 has a subgroup which is not normal.

[30]

2. (a) Find a subgroup H of order 2 in $\mathbb{Z}_{10}$.

   (b) Explain why H is normal in $\mathbb{Z}_{10}$.

   (c) Calculate the cosets of H in $\mathbb{Z}_{10}$ and write down the Cayley table for $\mathbb{Z}_{10}/H$.

   (d) Find a surjective homomorphism $\phi : \mathbb{Z}_{10} \to \mathbb{Z}_5$ with $\operatorname{Ker} \phi = H$. Deduce that $\mathbb{Z}_{10}/H \cong \mathbb{Z}_5$.

[20]

3. Let G be the rotational symmetry group of a cube. We have seen at lecture that $|G| = 24$. Consider the action of G on the set X where

   (a) $X =$ the set of all faces of the cube.

   (b) $X =$ the set of all edges of the cube.

   In each case, find the orbit and the stabilizer of an element in X and verify the Orbit-Stabilizer theorem.

[20]

4. How many different cubes can be constructed by

   (a) painting each edge of a cube red or blue?

   (b) painting each face of a cube red, yellow or blue?

[30]