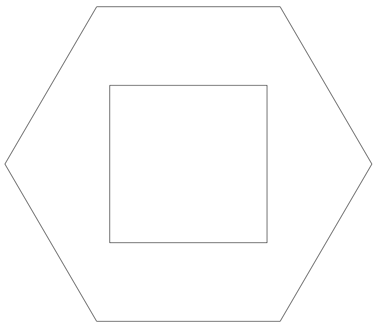
MA3615 Groups and Symmetry

Exercise Sheet 3: Subgroups, Cosets and Lagrange's theorem

1. (a) Show that $\mathbb{Z}_8$ has a subgroup of order 4 and a subgroup of order 2.
   (b) More generally, if n is divisible by m prove that $\mathbb{Z}_n$ has a subgroup of order m.
   (c) Find the left and right cosets of $\{0, 4\}$ in $\mathbb{Z}_8$.
2. Find all subgroups of S_3.
3. Describe the subgroups of $(\mathbb{Z}, +)$ generated by
   (a) 5
   (b) 4 and 6
   (c) 2 and 3
4. Show that the group $\mathbb{Z}_7^* = \{1, 2, 3, 4, 5, 6\}$ with multiplication modulo 7 is cyclic and find all the elements $n \in \mathbb{Z}_7^*$ such that $\langle n \rangle = \mathbb{Z}_7^*$.
5. Find the symmetry group G of the plane figure consisting of a square and a regular hexagon having the same centre and having four parallel sides (see picture below). How does G compare with the group D_8 (resp. D_{12}) of all symmetries of the square (resp. of the regular hexagon)? Can you find a 'known' group isomorphic to G?



6. Show that the intersection of two subgroups of a group G is also a subgroup of G.
7. Let $D_8 = \{e, r, r^2, r^3, s, rs, r^2s, r^3s\}$ be the dihedral group of order 8 (group of symmetries of a square). Find the left and right cosets of the subgroup H in D_8 where
   (a) $H = \langle s \rangle$
   (b) $H = \langle r \rangle$
8. Show that if H is a subgroup of S_4 with $|H| > 8$ then $|H| \geq 12$.