# MA3615 Groups and Symmetry

## Exercise Sheet 5: Group action, orbits, stabilizer, Burnside Counting Theorem

1. Let $G$ be the subgroup of $S_8$ generated by $(1,2,3)(4,5)$ and $(7,8)$. Then $G$ acts on the set $X = \{1,2,3,4,5,6,7,8\}$. Calculate the orbit and the stabilizer of each element in $X$ and verify the Orbit-Stabilizer Theorem in each case.

2. Calculate the order of the group of all rotational symmetries of the icosahedron. (The icosahedron has 20 faces consisting of equilateral triangles and 12 vertices, 5 faces meet at each vertex).

3. Consider the action of the group $D_{12}$ (all symmetries of a regular hexagon) on the set $X$ where

   (a) $X =$ the set of sides of the regular hexagon.

   (b) $X =$ the set of diagonals linking opposite vertices of the hexagon.

   In each case, determine whether the action is faithful. Also find the stabilizer in $D_{12}$ of a side (resp. a diagonal) of the hexagon.

4. Let $G$ be the group of all rotational symmetries of a cube. Let $H$ be the subgroup generated by a rotation by $\frac{\pi}{2}$ around an axis through the centre of opposite faces. Then $H$ acts naturally on the set $X$ of edges of the cube. Determine the $H$-orbits on $X$.

   Same question with $H$ being the subgroup generated by a rotation around one of the main diagonal the cube.

5. How many different necklaces can be made from seven black beads and three white beads.

6. Suppose it is proposed to manufacture ID cards from plastic squares, marked with a $3 \times 3$ grid on both sides and punched with two holes (see examples in the picture below). How many different ID cards can be produced in this way?