MA3615 Groups and Symmetry: Revision Sheet

Here is a list of the material (from your lecture notes, exercise sheet, test and coursework) that you are expected to know for the exam.

Chapter 1: Groups: Definition, Examples and First properties

- **Theory:**
  Define a group, an abelian group, the order of a group, an isomorphism between two groups, the order of an element in a group.
  Describe the following groups: C_n, D_{2n}, S_n, A_n.

- **Exercises:**
  -All of Exercise Sheet 1.
  -All of Exercise Sheet 2, except for the last part of question 7 (groups of order 4).
  -All questions from the Test.

Chapter 2: Subgroups, Cosets and Lagrange's theorem

- **Theory:**
  Define a subgroup H of a group G, a subgroup generated by a subset X, left/right cosets.
  State Lagrange's theorem (Theorem 2.7) and its consequences (Corollaries 2.8 and 2.9)

- **Exercises:**
  -All of Exercise Sheet 3.
  -Coursework qu.1(a)-(d).

Chapter 3: Normal subgroups, Quotient groups and Homomorphisms

- **Theory:**
  Define a normal subgroup (two equivalent definitions), a quotient group, a homomorphism from one group to another, the kernel and the image of a homomorphism. Prove that the kernel is a normal subgroup and the image is a subgroup.
  State the 1st isomorphism theorem (Theorem 3.5), use it to identify some quotient groups (ex. $\mathbb{Z}/7\mathbb{Z} \cong \mathbb{Z}_7$)

- **Exercises:**
  -All of Exercise Sheet 4, except for question 2.
  -Coursework qu.1(e)(f) and qu.2.

Chapter 4: Groups acting on sets

- **Theory:**
  Define what is meant by a group G acting on a set X, the G-orbit of an element $x \in X$, the stabilizer G_x of an element $x \in X$. prove that G_x is a subgroup of G.
  State the Orbit-Stabilizer theorem.
  Use the Orbit-Stabilizer theorem to calculate the order of some groups (for example the rotational symmetry group of the tetrahedron or the cube).
  State Burnside Counting theorem, apply it to count the number of distinguishable objects (ex. dice, painted triangles).

- **Exercises:**
  -All of Exercise Sheet 5.
  -Coursework qu. 3,4.

Chapter 5: Finite rotation groups

- **Theory:**
  Describe all rotational symmetries of a tetrahedron, a cube, an octahedron.
  List the 5 Platonic solids.
  Prove that the rotational symmetry group of the tetrahedron (resp. cube) is isomorphic to A_4 (resp. S_4) using the action of the group on the set of vertices (resp. the set of main diagonals).
  State (without proof) what the rotational symmetry groups of the other Platonic solids are.
  Define the orthogonal group and the special orthogonal group.
  State the classification of all finite symmetry group of 2D figures (Theorem 5.2.1).
  State the classification of all finite rotational symmetry group of 3D figures (Theorem 5.2.2).
  Use the classification theorem to work out symmetry groups of some given figures (ex. football).

- **Exercises:**
  -Exercise Sheet 6, qu.1-2