MA3615: Groups and Symmetry Mock Exam
Sketch solutions

1. (a) i. Not a group (justify)
   ii. Not a group (justify)
   iii. It is a group (justify)

   (b) bookwork

   (c) $\mathbb{Z}_4$ and C_4 (all rotational symmetries of a 4-pyramid) are isomorphic (give explicit isomorphism).
   But these are not isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$ (justify).

2. (a) bookwork

   (b) i. order 6
   ii. order 4

   (c) i. element of oder 1: e.
   elements of order 2: s, rs, r^2s.
   elements of order 3: r, r^2.

   ii.

	u	v	w	x	y	z
u	u	v	w	x	y	z
v	v	u	y	z	w	x
w	w	z	u	y	x	v
x	x	y	z	u	v	w
y	y	x	v	w	z	u
z	z	w	x	v	u	y

3. (a) bookwork.
   Check (S1)(S2)(S3) for H_1 and H_2.

   (b) Bookwork.
   For H_1 in S_3 find right and left cosets and check that they coincide.
   For H_2 in S_3 either show that left and right cosets do not coincide or find $g \in S_3$ and $h \in H_2$ with $ghg^{-1} \notin H_2$.
   H_1 and H_2 are not normal in S_4. The easiest way to show that is to find in each case a $g \in S_4$ and a $h \in H_i$ with $ghg^{-1} \notin H_i$. (can also show that left and right cosets do not coincide but it is much longer).

   (c) H_1, $(1,2)H_1 = \{(1,2),(1,3),(2,3)\}$. Write down Cayley table for S_3/H_1. Explicit isomorphism with $\mathbb{Z}_2$ given by $\phi(H_1) = 0$, $\phi((1,2)H_1) = 1$.

   (d) See exercise sheet 4, question 6.

4. (a) bookwork

   (b) bookwork

   (c) 15. (Add justification using Burnside counting theorem.)

5. (a) bookwork
   (b) check (S1)(S2)(S3).
   (c) bookwork
   (d) i. Take for example action of G on the set X of dotted edges. Then using Orbit-Stabilizer theorem we get that $|G| = 4 \times 2 = 8$. (give details).
       ii. bookwork.
       iii. From classification, $G \cong C_8$ or D_8. Rule out C_8 as G is not abelian (or by considering order of elements in G).