Fractals: Measuring Complexity

3rd year project 2007/08

Supervisor: Olalla Castro Alvaredo

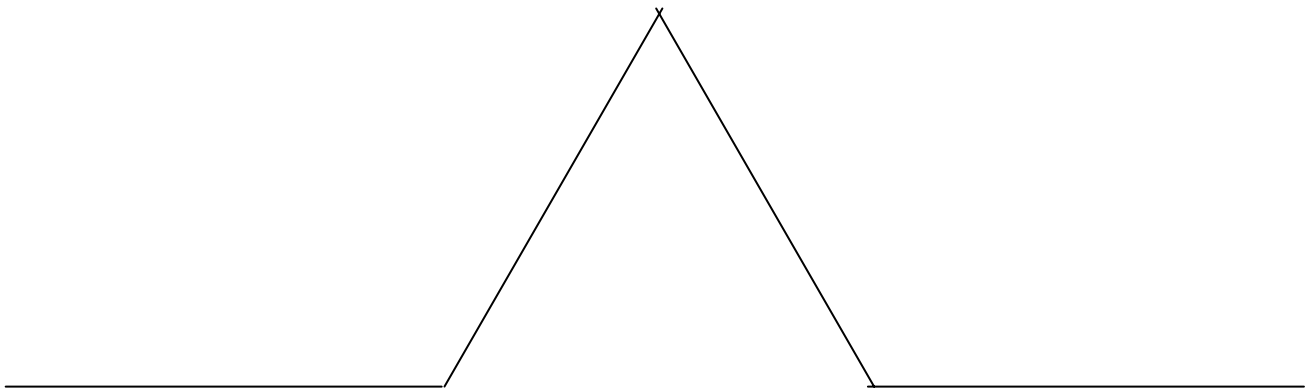
- The first objective of this project is to familiarise yourself with the main properties of some mathematical objects called fractals.
- I will give you some photocopies from chapter 11 of the book *«Non linear dynamics and chaos by Stephen Strogatz»* but there are many other books you can consult in the library and also a lot of material in the internet.
- Most books on dynamical systems or chaos will have a section on fractals (have a look in our library!)
- Once you have read a bit about fractals I would like you to try to answer the following questions:

- Fractals are typically described as being self-similar or quasi self-similar. What does this mean?
- Are there any fractals really existing in nature or are they purely mathematical objects? If you think that fractals really exist in nature, can you give some examples?
- What is the fractal dimension? Do you know any examples of fractal dimensions?
- Many fractals can be constructed by means of iterative or recursive procedures. How would you describe such procedures?

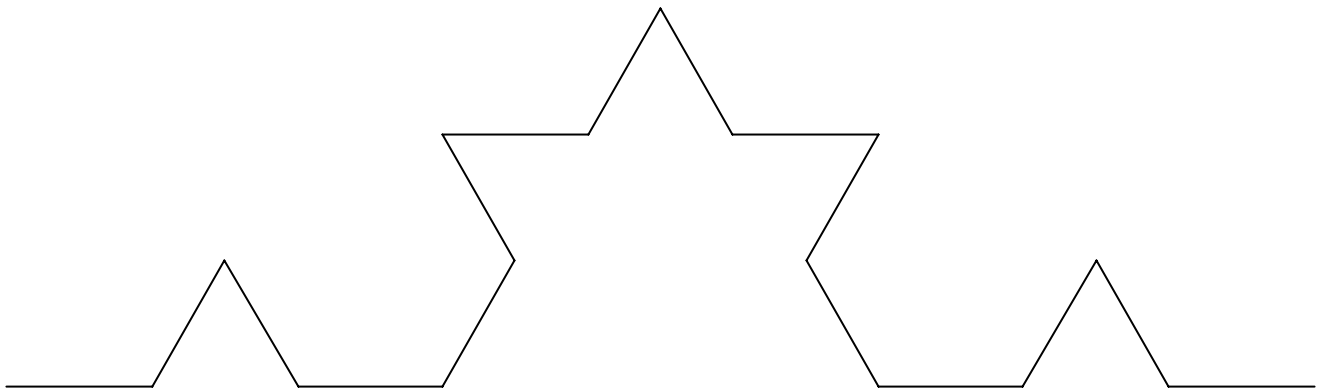
- Now I would like you concentrate on one particular fractal: *the Koch curve*.This is a typical example of a fractal which is constructed by means of an iterative procedure.
- Iterative procedures are usually easy to program. Could you write a computer program that generates the Koch curve?
- Even if you do not know much programmation try to figure out a recursive procedure such that given the two end points of the initial segment it would produce all end points of the 4 segments that are generated after one interation.

The Koch curve after 0 iterations

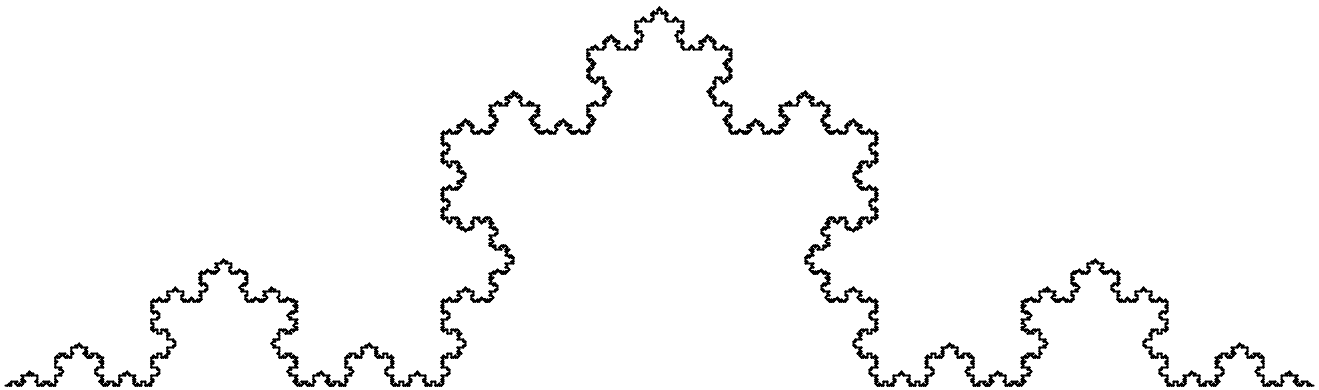
The Koch curve after 1 iteration



The Koch curve after 2 iterations



The Koch curve after many iterations



- Why do you think that the Koch curve does not have dimension 1? What makes it different from other curves?
- What is the length of the Koch curve? To answer this try computing the length of the curve obtained after 1 and 2 iterations and then think what happens if we do infinite iterations.
- What is a countable set? Hence, what is an uncountable set?
- The Koch curve can be constructed by joining together a certain set of points. How many points are there in that set?
- Do you think that this set is countable?

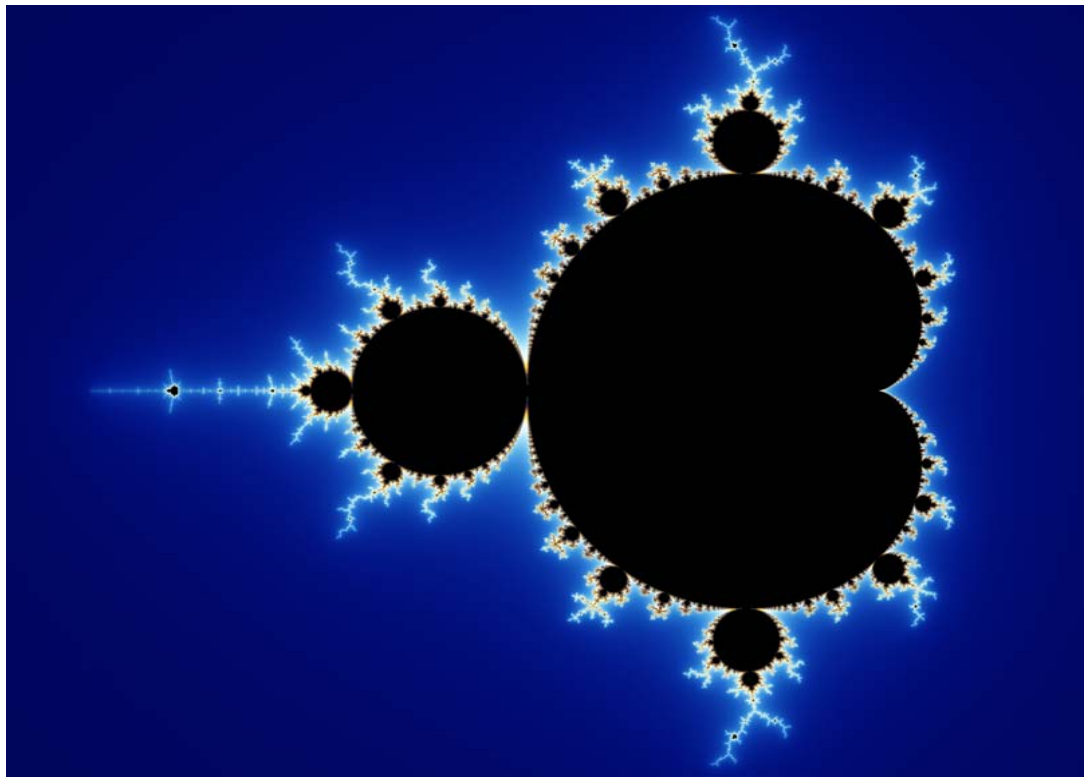
- The fractal dimension of the Koch curve can be obtained by computing the so-called box counting dimension. How is it defined?
- Compute the box counting dimension of the Koch curve.
- If you are good at programming, try to write a program that computes the box counting dimension of a given set of points.

- There are two types of fractals which are particularly famous: *The Mandelbrot Set and the Julia Sets.*
- They are closely related to each other and they are also constructed by means of an iterative procedure.
- In fact they are both related to the quadratic iterative map:

$$Z_{n+1}=(Z_n)^2 + c,$$

where c is a complex number. A picture of the Mandelbrot set is given in the next page!

- These fractals are more difficult to generate in a computer, but if you are good at programming you could try to generate one example.
- Describe the main properties of at least one of these two fractal types and explain how it is constructed.



Picture taken from:

http://en.wikipedia.org/wiki/Image:Mandel_zoom_00_mandelbrot_set.jpg