Dynamical Systems: Coursework 1 | T2

Student's Name:...

Instructions: For question 1, each wrong answer will contribute -5 points. For all other questions tick only one box. For questions 2,3,4 and 5, ticking more than one box or the wrong box will result in zero marks.

Q1 [20 points] Consider the following 1-dimensional dynamical system

$$\frac{dy}{dx} = \sin(y) - \cos(y)$$

Tick all the boxes which provide correct statements about the equation above

☑ The system has one fixed point in the region $\pi \leq y \leq 2\pi$

☐ The system is linear

☑ The fixed points of the system are of the form $y = \frac{\pi}{4} + n\pi$ with $n = 0, \pm1, \pm2....$

☐ The system has a fixed point at $y = \frac{3\pi}{2}$

☑ The system has infinitely many fixed points

☑ There is a fixed point at $y = -\frac{3\pi}{4}$ and it is an attractor

Q2 [20 points] Identify the phase diagram of the system of question 1 in the region $-2\pi \leq y \leq 2\pi$

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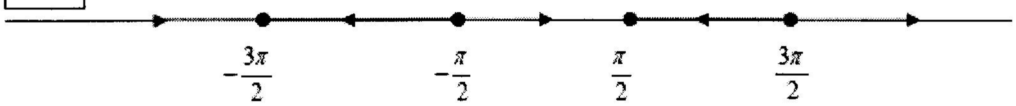

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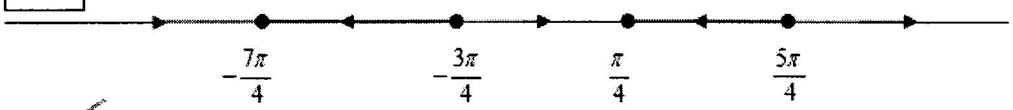

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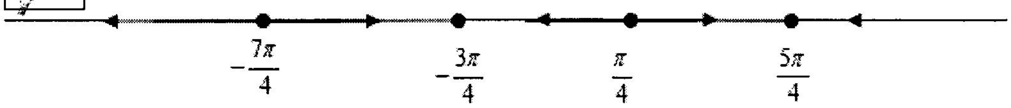

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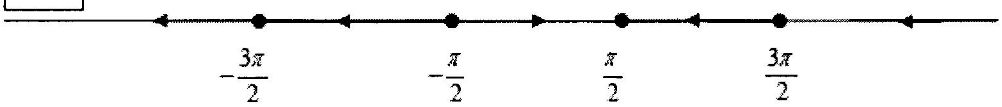


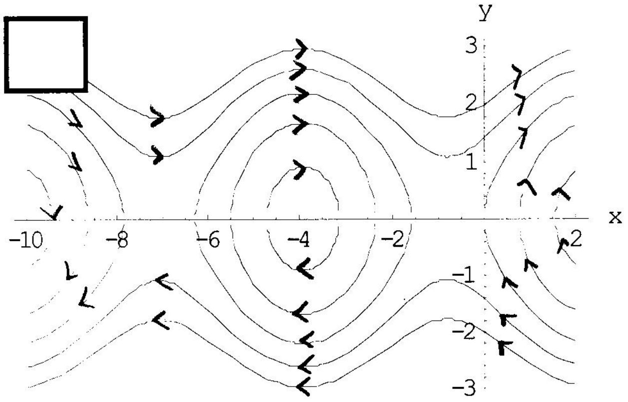
Q3 [20 points] Consider again the equation of question 1. The equation has a fixed point at y=a, with $0 < a \le \frac{\pi}{2}$. Find the value of a. Thus select the option below which gives the solution to the linearized version of the equation of question 1 about the fixed point a, with initial condition y=0 for x=2.

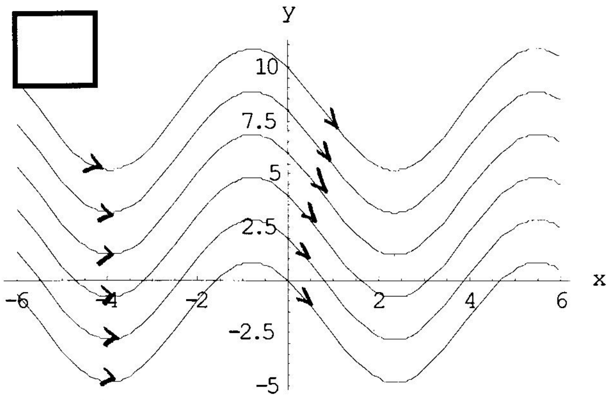
☐ $y = -\frac{\pi}{2} + \frac{\pi}{2} e^{x-2}$ ☑ $y = \frac{\pi}{4} - \frac{\pi}{4} e^{\sqrt{2}(x-2)}$ ☐ $y = \frac{\pi}{4} - \frac{\pi}{4} e^{x-2}$ ☐ $y = \frac{\pi}{2} - \frac{\pi}{2} e^{\sqrt{2}(x-2)}$

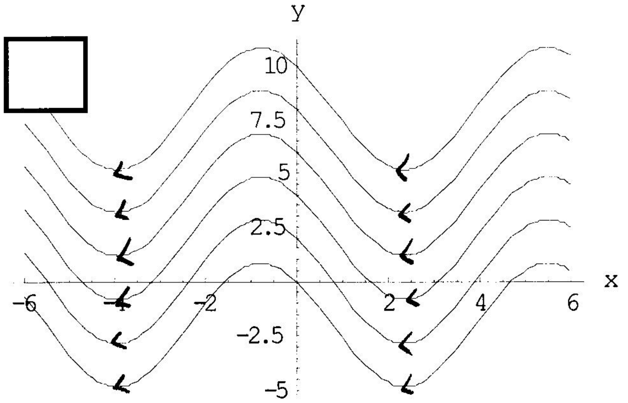
Q4 [20 points] Consider the 2-dimensional dynamical system

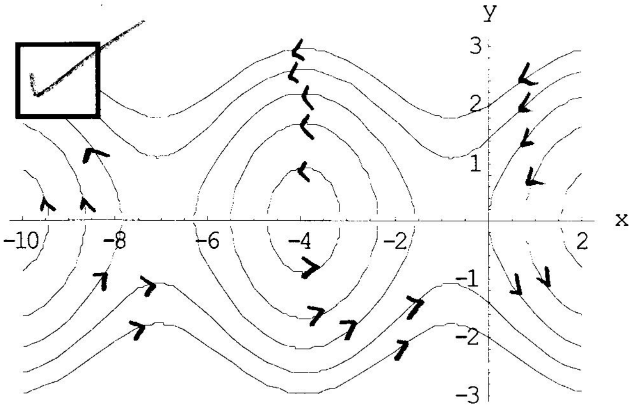
$$\frac{dy}{dt} = \sin(x) + \cos(x) \quad \text{and} \quad \frac{dx}{dt} = -y$$

Identify the phase diagram of this system of equations









Q5 [20 points] Consider the first order differential equation $\frac{dy}{dx} - \frac{y}{x} = \log(x)$. Identify which of the functions below is a particular solution to this equation

☐ $y = \frac{x}{2}(1 + \log(x))$ ☐ $y = \frac{x}{2}(1 - \log(x)^2)$

☐ $y = -\frac{x}{2}\log(x)^2$ ☑ $y = \frac{x}{2}(5 + \log(x)^2)$