Student's Name:.....

Instructions: For question 1, each wrong answer will contribute -5 points. For all other questions tick only one box. For questions 2,3,4 and 5, ticking more than one box or the wrong box will result in zero marks.

Q1 [20 points] Consider the following 1-dimensional dynamical system

$$\frac{dy}{dx} = -\sin(y) + \cos(y)$$

Tick all the boxes which provide correct statements about the equation above

☑ The system has two fixed points in the region $-\pi \le y \le \pi$

☑ The system is autonomous

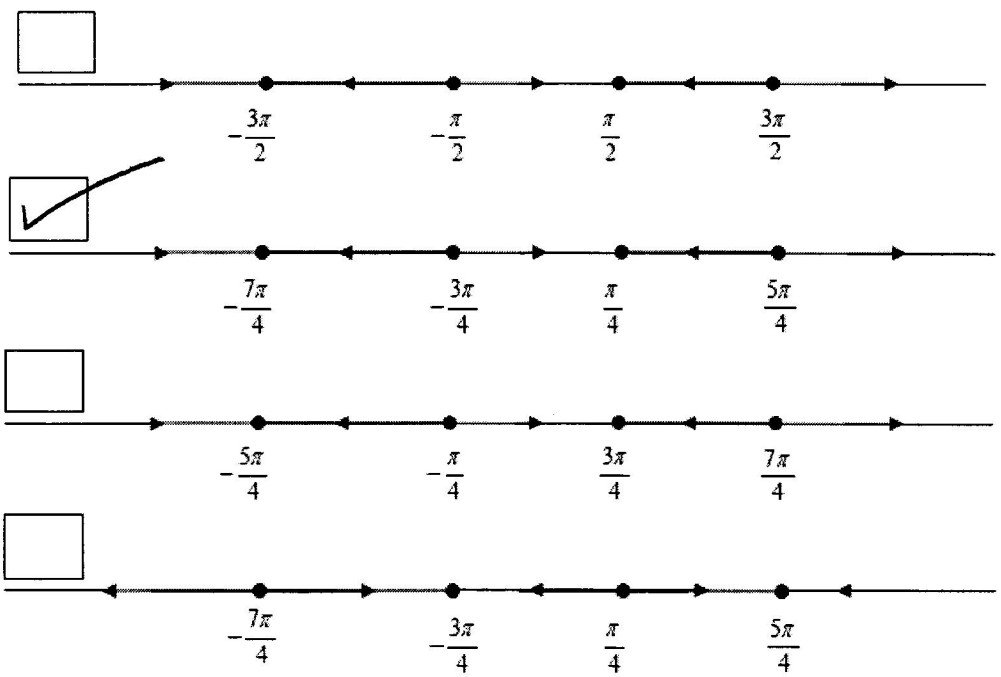
☑ The fixed points of the system are of the form $y = \frac{\pi}{4} + n\pi$ with $n = 0, \pm 1, \pm 2....$

☐ The system has a fixed point at $y = \frac{7\pi}{4}$

☑ The system has infinitely many fixed points

☐ There is a fixed point at $y = -\frac{7\pi}{4}$ and it is a repellor

Q2 [20 points] Identify the phase diagram of the system of question 1 in the region $-2\pi \le y \le 2\pi$



Q3 [20 points] Consider again the equation of question 1. The equation has a fixed point at y=a,with $\pi < a \leq \frac{3\pi}{2}$. Find the value of a. Thus select the option below which gives the solution to the linearized version of the equation of question 1 about the fixed point a, with initial condition y=0 for x=-1.

☑ $y = \frac{5\pi}{4} - \frac{5\pi}{4} e^{\sqrt{2}(x+1)}$ ☐ $y = \frac{3\pi}{2} - \frac{3\pi}{2} e^{x+1}$ ☐ $y = \frac{5\pi}{4} - \frac{5\pi}{4} e^{-\sqrt{2}(x+1)}$ ☐ $y = \frac{3\pi}{2} - \frac{3\pi}{2} e^{-\sqrt{2}(x+1)}$

Q4 [20 points] Consider the 2-dimensional dynamical system

$$\frac{dy}{dt} = -\sin(x) - \cos(x) \quad \text{and} \quad \frac{dx}{dt} = y$$

Identify the phase diagram of this system of equations

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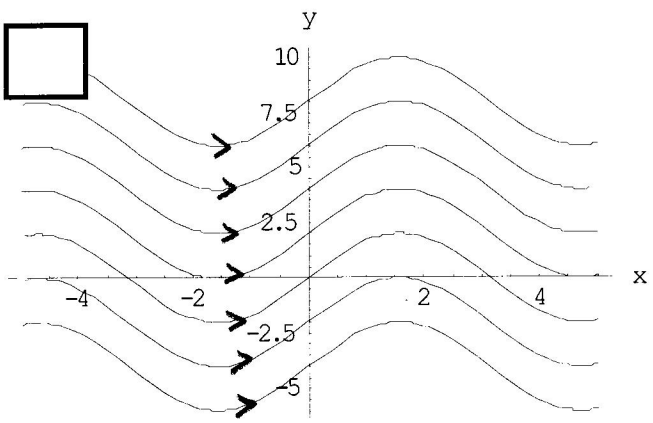

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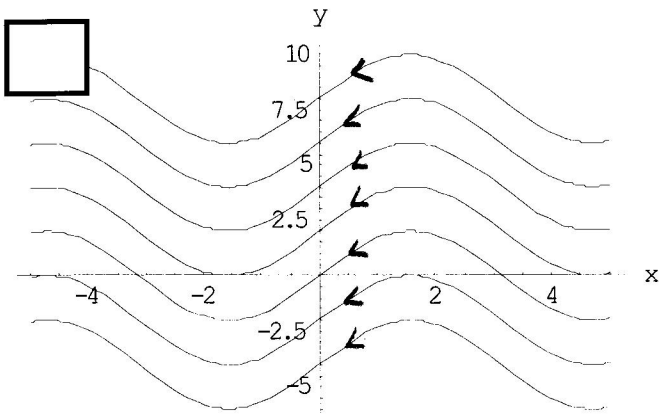

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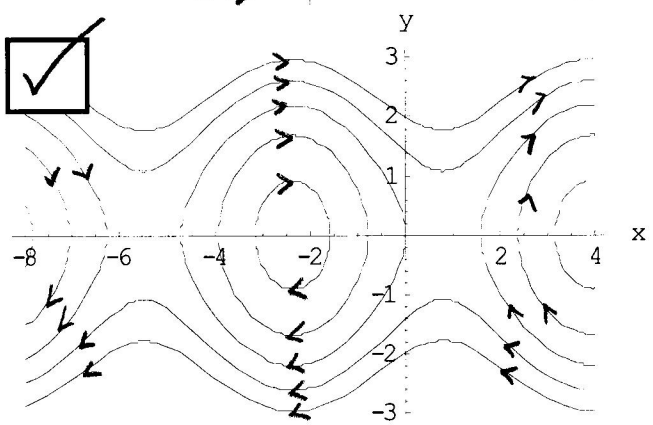

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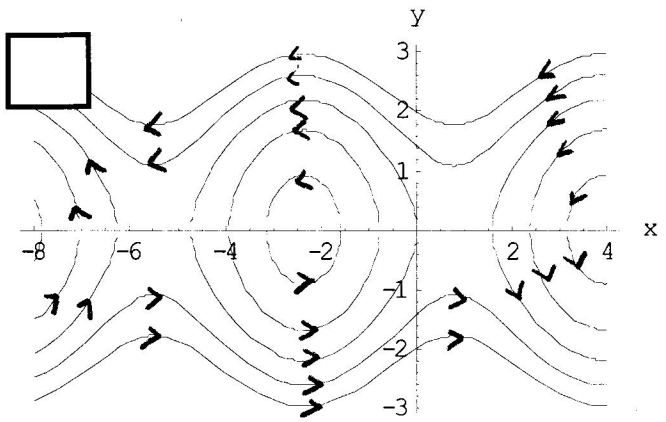

Q5 [20 points] Consider the first order differential equation $\frac{dy}{dx} + \frac{y}{x} = 1 + \log(x)$. Identify which of the functions below is a particular solution to this equation

☑ $y = \frac{x}{4}(1 + 2\log(x))$ ☐ $y = \frac{x}{4}(2\log(x) - 1)$

☐ $y = 1 + \frac{x}{4}\log(x)$ ☐ $y = \frac{2}{x} + \frac{x}{4}(1 + \log(x))$