Sheet 1: differential equations and phase diagrams

1. Use the method of *separation of variables* to find the solution of following differential equations for the given initial condition:

$$\begin{aligned}
\frac{dy}{dx} &= 1+y^2, \qquad \text{with} \qquad y(0)=1,\\
\frac{dy}{dx} &= x\cos^2 y, \qquad \text{with} \qquad y(0)=\frac{\pi}{4},\\
\frac{dy}{dx} &= y^2+2y-3, \qquad \text{with} \qquad y(0)=-1.
\end{aligned}$$

2. Use the method of *integrating factors* to find the solution of following differential equations for the given initial condition:

$$\begin{aligned}
\frac{dy}{dx} &= \cos x - y\tan x, \qquad \text{with} \qquad y(0)=1,\\
\frac{dy}{dx} &= e^x - 3y, \qquad \text{with} \qquad y(0)=\frac{1}{2},\\
\frac{dy}{dx} &= \cos x - y\cot x, \qquad \text{with} \qquad y(\frac{\pi}{2})=1.
\end{aligned}$$

3. Sketch a phase diagram for the first order nonlinear equation,

$$\frac{dy}{dt} = y(1-y^2).$$

4. Construct the velocity/vector diagram for the following equations using the Excel software available from the module's web page[1]. Run the programme to produce diagrams in the range $-2 \le x_1, x_2 \le 2$. In each case try to construct from this a rough phase diagram:

$$\frac{dx}{dt} = ax+by, \quad \frac{dy}{dt} = cx+dy,$$

where the constants a, b, c and d take the following three sets of values: $a=13, b=9, c=-18, d=-14$ or $a=5, b=4, c=-10, d=-7$ or $a=3, b=1, c=-1, d=1$.

[1] http://www.staff.city.ac.uk/o.castro-alvaredo/dynamical/dynamical.html