

## End of term test

**Attempt all questions**

**90 minutes**

*This is an Open Book test. Do NOT use a calculator.*

*All results from DERIVE and any associated working should be shown in your answer book.*

### Question 1

- a Plot the graph of

$$y = f(x) = x^3 - 13x^2 + 22x - 10,$$

showing a rough sketch in your answer book.

- b Use your graph of  $f(x)$  and the on-screen cross to write down *all* the roots of this equation. Also find the coordinates of any local maxima and minima. Give your answers correct to *two* decimal places.
- c By either rearranging the function or by Newton's method, write down an iterative scheme for finding the roots of this equation.
- d Use your iterative method to calculate all the roots of the equation to four decimal places. For each root give your initial guess and the number of iterations required for convergence.

### Question 2

- a Write out the equations

$$\begin{aligned} x + y + z &= 1, \\ 2x + 4y + 3z &= 3, \\ 2x + 6y + 3z &= 4, \end{aligned}$$

in matrix form  $A\mathbf{x} = \mathbf{b}$ .

- b Use DERIVE to calculate  $A^{-1}$ .
- c Hence using DERIVE or otherwise calculate  $A^{-1}\mathbf{b}$  to obtain the solutions to the simultaneous equations.
- d What happens if the third equation is replaced by

$$2x + 6y + 4z = 4.$$

### Question 3

A manufacturer of lamps makes two sorts of lamps: a desk lamp and a standard lamp. Each desk lamp is sold for £10 more than its cost to make, while each standard lamp is sold for £15 more than its cost. It also cost £2400 a month to keep the lamp workshop running. All the lamps made are sold.

- a If  $x$  denotes the number of desk lamps produced each month and  $y$  the number of standard lamps produced each month, write down an expression for the profit  $P$  made each month by the lamp manufacturer.
- b There are two workers in the lamp workshop. The first assembles the lamps, taking 30 minutes to assemble a desk lamp and 20 minutes to assemble a standard lamp. She only works for 100 hours per month. The second worker paints the lamps, taking 10 minutes to paint a desk lamp and 20 minutes to paint a standard lamp. He only works for 80 hours each month. From this information write down two inequalities for  $x$  and  $y$  that must hold. What other inequalities must hold for  $x$  and  $y$ ?
- c Use DERIVE to plot a single feasible region for *all* of the inequalities in the problem (show a rough sketch in your answer book).
- d Use DERIVE to obtain the maximum profit, and the number of lamps of each type to be produced each month to generate this profit.